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Note

Equilibrium selection in common-value second-price auctions

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A B S T R A C T

This note considers equilibrium selection in common-value second-price auctions with two bidders. We show that for each *ex post* equilibrium in continuous and undominated strategies, a sequence of "almost common-value" auctions can be constructed such that each of them possesses a unique undominated and continuous equilibrium and the corresponding sequence of equilibria converges to that *ex post* equilibrium. As an implication, no equilibrium selection of this model based on perturbations seems to be more convincing than others.

1. Introduction

The well-known linkage principle (Milgrom and Weber, 1982) in auction theory states that the expected revenue in the symmetric equilibrium of a second-price auction is no less than the expected revenue from a first-price auction. However, Milgrom (1981) uses a simple example to illustrate that there could be a continuum of *asymmetric* equilibria in common-value second-price auctions, which are not revenue-equivalent. In fact, it is easy to see that the seller's revenue in an asymmetric equilibrium can be very low. Thus, multiplicity of equilibria generates difficulties for revenue comparisons in common-value auctions. While in his original work Milgrom calls these asymmetric equilibria "strange," a subsequent study by Klemperer (1989) suggests that asymmetric equilibria may be the only reasonable ones in the sense that by giving a slight advantage to one bidder, almost all equilibria are "extreme" as the advantaged bidder wins the auction with probability one in any undominated and continuous equilibrium. Therefore, there seems to be no obvious reason to favor the symmetric equilibrium over asymmetric ones.

In response to this multiplicity problem, various studies have been devoted to selecting a particular equilibrium in the second-price auction by perturbing the model in different ways.[1] Parreiras (2006) perturbs the second-price auction format to a hybrid auction involving the winner paying the highest bid with a small probability and the second highest bid with the complementary probability. He shows that the hybrid auction generates at least as much revenue as the first-price auction when signals are affiliated, thereby providing a justification for the linkage principle. Abraham et al. (2012) define a notion of tremble-robust equilibrium based on the idea that there is a small probability that an additional bidder may be present in the auction and draws her bid according to a predetermined smooth distribution.[2] In a model with asymmetrically-informed bidders, they select the equilibrium that generates the lowest revenue for the seller. Cheng and Tan (2008) provide a

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[1] There is also an alternative approach to equilibrium selection in auctions based on iterated elimination of dominated strategies. See Harstad and Levin (1985) for an example.

[2] Equilibrium selection via the introduction of a noisy bidder is first considered by Hashimoto (2010) in a complete information generalized second-price auction.

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justification of the symmetric equilibrium by adding a small private-value component to the common-value model. Larson (2009) also considers private-value perturbations. He shows that by adding a private-value component that is independent of the common-value signals, asymmetric perturbations lead to selections of asymmetric equilibria under certain restrictions on the common-value component and signal distributions.

In contrast to the previous literature, this note provides a general analysis of equilibrium selection in pure common-value second-price auctions. For such auctions, we provide a negative conclusion to the approach of equilibrium selection based on payoff perturbations. In particular, we show that every increasing and continuous equilibrium can be selected by perturbing bidders' valuations in a certain manner. An implication of this result is that symmetric equilibria can only survive symmetric perturbations of payoffs. Similar results hold in second price auctions with more than two bidders and in English auctions more generally.

While the main results apply to equilibria in monotone and continuous strategies, we also identify a class of equilibria in discontinuous and undominated strategies that may not be monotone in common-value second-price auctions.[3] However, we show that all those discontinuous equilibria are fragile to the introduction of a noisy bid. In contrast, all equilibria in continuous and undominated strategies are robust to this perturbation, thereby justifying our focus on the continuous equilibria.

2. A common-value second-price auction with two bidders

Consider a pure common-value auction with two bidders. There is a single object for sale and two risk-neutral bidders compete for the object via a sealed-bid second price auction. The value V of the object is the same to both bidders. Prior to submitting bids, each bidder receives a private signal that partially reveals the value of the object. For each $i \in \{1, 2\}$, let $\tilde{s}_i$ denote bidder i's private signal. Assume that $(\tilde{s}_1, \tilde{s}_2)$ is drawn according to the cumulative distribution function F with support $[0, 1] \times [0, 1]$. For each $i, j \in \{1, 2\}$ and $i \neq j$, let $F_i(\cdot|s_j)$ denote the distribution of s_i conditional on bidder j's signal realization s_j. Assume that $F_i(\cdot|s_j)$ admits a density function $f_i(\cdot|s_j)$ that is strictly positive on $[0, 1]$. The expected value of the object conditional on the signal pair (s_1, s_2) is given by $v(s_1, s_2) \equiv \mathbb{E}[V|s_1, s_2]$. Finally, assume that v is continuously differentiable and strictly increasing in each s_i.

A pure strategy for bidder i is a map $\beta_i : [0, 1] \to \mathbb{R}$, which determines her bid for any signal. We will consider equilibria in undominated pure strategies. Since there are two bidders, this model is equivalent to an English (open ascending-price) auction. It is well-known that this pure common-value auction has multiple equilibria.[4] The following class of undominated *ex post* equilibria is identified by Milgrom (1981).

Lemma 2.1. *For every strictly increasing and onto function $h : [0, 1] \to [0, 1]$, the strategy profile $\beta_1(s_1) = v(s_1, h^{-1}(s_1))$ and $\beta_2(s_2) = v(h(s_2), s_2)$ is an undominated* ex post *equilibrium. Furthermore, all undominated* ex post *equilibria in continuous and increasing strategies are of this form.*

Proof. See Milgrom (1981) and Bikhchandani and Riley (1991). □

Note that the seller's revenue in an asymmetric equilibrium can be very low. For example, consider the function $h(s) = s^\alpha$ where α is a constant. For large α, bidder 1's bids are close to $v(s_1, 0)$ with high probability. Since the losing bid determines revenue in a second-price auction, the seller's expected revenue is close to $\mathbb{E}[v(s_1, 0)]$ in this asymmetric equilibrium.

Unlike prior studies that select a particular equilibrium in second-price auctions (especially the symmetric equilibrium), in the next section we obtain a negative answer to equilibrium selection based on perturbations. Our results suggest that all these asymmetric equilibria are equally convincing.

3. Equilibrium selection by private-value perturbations

Consider the following class of "almost common-value" second-price auctions. Let $\mathcal{H}$ denote the collection of all strictly increasing and continuous functions that map $[0, 1]$ onto $[0, 1]$. For each $h \in \mathcal{H}$ and each $\varepsilon \in (0, 1)$, define the corresponding second-price auction $\Gamma^{\varepsilon,h}$ by perturbing the *ex post* payoff functions of both bidders to

$$\tilde{v}_1^\varepsilon(s_1, s_2) = \varepsilon s_1 + (1 - \varepsilon) v(s_1, s_2),$$
$$\tilde{v}_2^\varepsilon(s_1, s_2) = \varepsilon h(s_2) + (1 - \varepsilon) v(s_1, s_2).$$

Suppose that bidder 2 follows a monotone bidding function β_2, then bidder 1 with signal s_1 will bid b in order to maximize

[3] Birulin (2003) points out that there exist undominated *ex post* equilibria in discontinuous strategies when the auction admits an efficient *ex post* equilibrium.

[4] Milgrom (1981) first points out the multiplicity of *ex post* equilibria in common-value second-price auctions. Bikhchandani and Riley (1991) argue that there is a much larger class of perfect Bayesian equilibria in English auctions with more than two bidders.