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Asymmetric parametric division rules

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ABSTRACT

We describe and characterize the family of asymmetric parametric division rules for the adjudication of conflicting claims on a divisible homogeneous good. As part of the characterization, we present two novel axioms which restrict how a division rule indirectly allocates between different versions of the same claimant. We also show that such division rules can alternately be represented as the maximization of an additively separable social welfare function.

1. Introduction

When a firm goes bankrupt, how should its liquidated value be distributed among creditors? How should an estate be divided among heirs when more is promised in a will than is available? How should the cost of a project be shared among the group of beneficiaries? What is a fair way to tax citizens?

The first two questions are known as *conflicting claims problems*, or simply *claims problems*. The question is how to distribute fairly some good when there is an insufficient amount of the good to satisfy all claims on it. A solution to the problem is a *division rule*, which assigns to every claims problem an allocation, or *award*, to the claimants. We classify division rules by the axioms they satisfy. The problem is as old as human history, and specific examples with proposed awards are even offered in the Talmud. O'Neill (1982) was the first to formalize the problem, and since then numerous rules and axioms have been proposed.[1] Formally, the claims problem is identical to the problems of cost-sharing and fair taxation, though we will primarily use the claims interpretation.

We characterize a family of division rules which we call (*asymmetric*) *parametric rules*, a generalization of Young's (1987) class of symmetric parametric rules, and which was first introduced by Thomson (2006, p. 99). Parametric rules divide as follows: There is a collection of continuous monotone functions $\{f_i(c_i, \cdot)\}$ indexed by all possible claimants i and all possible claims c_i. Each $f_i(c_i, \cdot)$ represents a schedule of possible awards that specifies how much claimant i with claim c_i is awarded over all possible values of a parameter. Thus for parameter value λ, the amount awarded to i with claim c_i is $f_i(c_i, \lambda)$. For a given claims problem, a common parameter is chosen for all claimants so that all of the good is distributed. Intuitively, one can think of the parameter as some sort of measure of fairness, and the function $f_i(c_i, \cdot)$ is then simply the translation of

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[1] See Thomson (2003) for a survey.

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this measure to an award. The choice of a common parameter implies that the claimants are being treated equitably with respect to this standard of fairness.

As part of our characterization of parametric rules, we use two axioms that are widely used in the literature: Consistency and Resource Monotonicity. Consistency states that if a division rule chooses an allocation for a group of claimants, then it should not choose to reallocate the awards of any subgroup when considered as a separate problem. Resource Monotonicity states that if the amount to be divided increases, then no claimant's award should decrease.

Another common axiom is Symmetry, which states that claimants with equal claims should receive equal awards. Obviously, parametric rules do *not* generally satisfy Symmetry. We study asymmetric division rules for normative and positive reasons. For reasons of fairness, we may not want a rule to be symmetric. As an informal example, parents may treat their children differently because they recognize each child is different and has different needs, even though the children might protest that they are being treated asymmetrically. More generally, there may be considerations outside of the model (e.g. rights, needs, history) that require a rule to treat claimants asymmetrically. Thus, depending on the context of the problem, fairness may require a rule to be asymmetric. From a descriptive stand point, there are many rules, especially real-life rules, that are not symmetric. For example, U.S. bankruptcy law stipulates that when a firm goes bankrupt, taxes owed to the federal government must be paid before other claims.[2] The forming of a queue (e.g., to purchase tickets to a popular concert or sporting event, to withdraw money during a bank run) is another example. Thus by not imposing Symmetry we are able to better understand these common division rules.

We introduce two novel axioms as part of our characterization. To understand the role these axioms play, observe that an asymmetric parametric rule could potentially allocate intrapersonally. That is, one could observe how a rule might allocate between different versions of the same claimant by comparing, say, $f_i(c_i, \cdot)$ to $f_i(c'_i, \cdot)$, where c_i and c'_i are different claims i could have. However in the traditional formulation of a claims problem, a division rule does not allocate intrapersonally, as a problem cannot have the same claimant with two different claims. But a division rule does *indirectly* allocate intrapersonally. That is, one could observe how a rule allocates between i when his claim is c_i and a second "go-between" claimant, j, with claim c_j, and then compare that to how the rule allocates between i with claim c'_i and j with claim c_j. This would reveal how the rule allocates intrapersonally.

Our two axioms put restrictions on how the rule allocates intrapersonally.[3] The first axiom, Intrapersonal Consistency, states that how the rule indirectly allocates between different versions of claimant i will not change when the go-between's claim c_j changes. However, it may be that there are some intrapersonal allocations that cannot be compared, meaning there is no go-between claimant that would reveal how the rule intrapersonally allocates. The second axiom, Non-comparability Continuity in Claims at Priority Points (or N-Continuity for short), states that the non-comparability of two allocations is a continuous relation with respect to small changes in the claim.

The axioms that characterize parametric rules are Continuity, N-Continuity, a weaker version of Consistency known as Bilateral Consistency, Intrapersonal Consistency, and Resource Monotonicity. We also show that if Resource Monotonicity is strengthened to Strict Resource Monotonicity, the resulting characterization can be derived without Intrapersonal Consistency and N-Continuity.

In his paper, Young also showed that there is a connection between a symmetric parametric rule and a rule that can be written as the result of maximizing an additively separable and symmetric social welfare function. We generalize this result as well for asymmetric rules. That is, we show that any parametric rule maximizes a strictly convex and additively separable social welfare function.

There is a growing literature on asymmetric division rules. Moulin (2000) derives a rich family of asymmetric rules that satisfy Consistency, as well as axioms not considered here, namely Upper Composition, Lower Composition, and Homogeneity. Chambers (2006) studies a similar family, though without imposing Homogeneity. Naumova (2002) characterizes an asymmetric version of Young's (1988) family of equal sacrifice rules. The key axioms there are Consistency, Upper Composition, and Strict Resource Monotonicity. Hokari and Thomson (2003) characterize a family of asymmetric rules which generalize the Talmud rule, and derive the consistent extensions of these rules. Ju et al. (2007) accommodate the US bankruptcy rule by expanding a claims problem to allow for multi-dimensional claims. However their focus is on rules that give no benefit to claimants who transfer claims between themselves (an axiom called Reallocation-proofness).

Kaminski (2006) also accommodates division rules like the US bankruptcy rule by expanding the definition of a claims problem, though he does this even more generally than Ju et al. (2007). Instead of a claim, each individual has a "type" (of which a claim may be a part). Though Symmetry is assumed, this is with respect to types, meaning claimants with the same type receive the same award. Thus, individuals with identical claims may receive different awards if their types differ in other respects.

Interestingly, Kaminski's results can be used to provide an alternative characterization of the family of asymmetric parametric rules that we consider here. We discuss this in more detail in the conclusion. But to summarize briefly, this requires defining a claims problem to include ones where one claimant appears multiple times with different claims. The advantage of this approach is that only "standard" axioms are needed. The disadvantage is that it uses a definition of a claims problem which is unrealistic. As a result, it hides the issue of intrapersonal allocation which we discussed earlier.

[2] See Kaminski (2000, 2006).

[3] Obviously when Symmetry is assumed, how a rule allocates intrapersonally is not an issue as this can be inferred from how the rule allocates interpersonally.