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Minimum cost spanning tree problems with indifferent agents

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A B S T R A C T

We consider an extension of minimum cost spanning tree (mcst) problems in which some agents do not need to be connected to the source, but might reduce the cost of others to do so. Even if the cost usually cannot be computed in polynomial time, we extend the characterization of the Kar solution (Kar, 2002) for classic mcst problems. It is obtained by adapting the Equal treatment property: if the cost of the edge between two agents changes, their cost shares are affected in the same manner if they have the same demand. If not, their changes are proportional to each other. We obtain a family of weighted Shapley values. Three interesting solutions in that family are characterized using stability, fairness and manipulation-proofness properties.

1. Introduction

Minimum cost spanning tree (mcst) problems study situations in which a group of agents need to connect to a source in order to obtain a good or service. There is a cost to be paid for each edge in the network that is used. That cost is independent of the number of agents that use that edge to connect to the source. There is a large literature on the associated cost-sharing problem, with the first use of game-theoretic tools appearing in Bird (1976). That article showed that the core is always non-empty and proposed a method, now known as the Bird solution, that is always in the core.

We propose an extension to the mcst model: instead of assuming that all agents desire to be connected to the source, we allow for a subset of agents to be indifferent to such a connection. While these agents have no demand, they still may have an impact on the cost of the project as their cooperation might allow other agents to connect to the source at a cheaper cost. We then have the non-trivial problem of potentially compensating these agents for their contributions to lowering cost.

This extension allows us to cover a wide range of problems. Obviously, if all agents want to be connected, we are back in the classic mcst problem. If only one agent wants to be connected, this turns out to be equivalent to a shortest path problem. The problem considered is also close to Steiner tree problems (see Hwang and Richards, 1992 for a review), in which some freely available nodes can be used by anyone. Steiner tree problems and the general problem considered here share the undesirable characteristics that the minimum cost of the project usually cannot be computed in polynomial time and that the core can be empty. Bergantinos et al. (2011) attempt to provide cost-sharing solutions to Steiner tree problems. The difference with our setup is that we do not consider these nodes as unoccupied, and we allow for compensation to their owners. If such compensations were disallowed, the problem would become a Steiner tree problem.[1]

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[1] In Trudeau (2012), there is a distinction between the common and the private property approaches. In the former, a coalition can use the location of any of its neighbors to connect to the source, while in the latter the coalition can only use the locations of its members.

Using the common property approach with indifferent agents would be equivalent to disallowing compensations to indifferent agents. Throughout the paper, we use the private property approach.

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The relevance of the extension is not only theoretical: the minimum cost spanning tree problem applies well to physical distribution networks of goods like water, gas or electricity. However, most of these projects involve states or cities that do not want the good (or are not its main destination) but that host part of the network. Conflicts with these agents are frequent, as the use of their territory usually requires compensation.[2]

The cooperation among agents that need to be connected to the source and agents that can provide them with cheaper connections shares similarities with the general technological cooperation models (Trudeau, 2009a; Bahel and Trudeau, 2013) in which agents provide an unspecified service (sharing patents, know-how, providing access to suppliers, etc.) that reduces the overall cost. These issues have been raised for the mcst problems, notably when discussing if it is acceptable to subsidize some agents, as the Kar solution does (Bergantinos and Vidal-Puga, 2007; Trudeau, 2012).

Three of the main cost sharing solutions in the minimum cost spanning tree literature (the Bird solution, the folk solution (Feltkamp et al., 1994; Bergantinos and Vidal-Puga, 2007) and the cycle-complete solution (Trudeau, 2012)) are designed to be in the core. Given that in our context the core can be empty, there is limited appeal in extending these methods to our context. We therefore focus on the Kar solution (Kar, 2002), which is the Shapley value of the stand-alone game.

Kar (2002) characterizes his solution with Group Independence, which implies that if there are never any gains for two groups to cooperate on the construction of the network, then we can compute the cost shares independently for each group. It also uses the property of Equal Treatment, which implies that if the cost of edge (i, j) goes down, the cost shares of agents i and j should change by the same amount. This notion is very natural as the edge that generates the saving is jointly owned by the two agents. If Group Independence can be extended easily to our context, it is not trivial to adapt Equal Treatment. It seems natural to keep the requirement if i and j are identical with respect to their desire to connect to the source, as that edge can be used to connect i or j to the source. If i wants to be connected but not j, it does not seem as natural to require them to have equal changes in their cost shares, as the agents play different roles. In particular, the edge can only be used to connect i. Therefore, we only require that the changes be proportional to each other. This is a consistency requirement, as this guarantees that the way we treat demanders versus non-demanders remains constant throughout. In particular, that treatment does not depend on the magnitude of the change in cost, the way it affects the optimal configuration or the identity of the agents involved.

When we adapt Equal Treatment to cases in which agents have different desire to connect to the source, if we require that a demander and a non-demander be both (strictly) affected by the change in cost of the edge between them, we characterize a family of weighted Shapley values. We then describe three interesting weighted Shapley values. The first one is obtained by requiring that when the cost of an edge between a demander and a non-demander decreases, both agents are affected in the same manner. The other two methods considered are limits of the characterized family of weighted Shapley values when the impact on one type of agents tends to zero.

We then introduce a series of properties to differentiate the three solutions. These properties are related to stability, the rent given to non-demanders and the non-manipulability of demands. The properties are used to characterize the three weighted Shapley values.

The paper is divided as follows: Section 2 describes the model and provides an example of how the optimal configuration is computed. Section 3 shows that the core can be empty. Section 4 describes and characterizes the family of methods obtained by extending Kar's characterization for mcst problems. Section 5 differentiates and characterizes three of those methods. As our model covers many different cases explored in the literature, links with these models are clarified in Section 6.

2. The model

In order to define mcst problems with indifferent agents, we first define classic mcst problems.

2.1. Minimum cost spanning tree problems

Let $\mathcal{N} = \{1, 2, \ldots, \}$ be the set of potential participants and $N \subseteq \mathcal{N}$ be the set of actual participants that need to be connected to the source, denoted by 0. Let $N_0 = N \cup \{0\}$. For any set $Z \subseteq \mathcal{N} \cup \{0\}$, define Z^p as the set of all non-ordered pairs (i, j) of elements of Z. In our context, any element (i, j) of Z^p represents the edge between i and j. We often write a generic element of Z^p, an edge, as e. Let $c = (c_e)_{e \in N_0^p}$ be a vector in $\mathbb{R}_+^{N_0^p}$ with c_e representing the cost of edge e. Let $\Gamma(N)$ be the set of all cost vectors when the set of agents is N, with $N \subseteq \mathcal{N}$. Let Γ be the set of all cost vectors, for all possible N. Since c assigns a cost to all edges e, we often abuse language and call c a cost matrix. A minimum cost spanning tree problem is a triple $(0, N, c)$. Since 0 does not change, we omit it in the following and simply identify a mcst problem as (N, c), with $N \subseteq \mathcal{N}$ and $c \in \Gamma(N)$.

A cycle p_{ll} is a set of $K \geqslant 3$ edges (i_k, i_{k+1}), with $k \in [0, K-1]$ and such that $i_0 = i_K = l$ and $i_1, \ldots, i_{K-1}$ distinct and different than l. A path p_{lm} between l and m is a set of K edges (i_k, i_{k+1}), with $k \in [0, K-1]$, containing no cycle and such that $i_0 = l$, $i_K = m$ and $i_1, \ldots, i_{K-1}$ distinct and different than l and m. A spanning tree is a non-orientated graph

[2] Negative externalities coming with the network are another reason for conflict. This aspect of the problem is not modeled here.