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Sharing the cost of redundant items

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ABSTRACT

We ask how to share the cost of finitely many public goods (items) among users with different needs: some smaller subsets of items are enough to serve the needs of each user, yet the cost of *all* items must be covered, even if this entails inefficiently paying for redundant items. Typical examples are network connectivity problems when an existing (possibly inefficient) network must be maintained.

We axiomatize a family cost ratios based on simple *liability indices*, one for each agent and for each item, measuring the relative worth of this item across agents, and generating cost allocation rules additive in costs.

1. Introduction

Consider a group of agents with different needs who must share the cost of several indivisible public goods (*items*). The needs of a user are met by certain combinations (subsets) of these goods, and the pattern of these subsets is arbitrary. This very general problem encompasses a variety of familiar fair division problems, including

- partners sharing a library of software licenses;
- cities sharing a set of antennas, routers, or other broadcasting devices geographically dispersed, so that each device reaches only a subset of cities;
- users of a network requiring certain connectivity between certain nodes of the graph, as in the minimal cost spanning tree problem, and its variant where some agents may require higher order connectivity, or its generalization where each agent needs to connect a possibly different set of nodes.

What makes the problem difficult is that the substitutability of the different items can differ greatly across agents: for instance one user of a network may be served by any path connecting "his" node to the source, while another user needs a path avoiding certain edges.

The familiar axiomatic approach to such cost sharing problems assumes that the agents provide a cheapest (efficient) subset of items serving all individual needs, and pays special attention to the cheapest cost of (hypothetically) covering only the needs of each conceivable subset of agents (the Stand Alone costs of these subsets). We assume instead that the set of

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items is exogeneously fixed, that it is enough to cover all needs, and typically includes redundant items, i.e., items that can be discarded without any service interruption, but that our agents must nevertheless pay for. This assumption is realistic when the items represent long-lived pieces of infrastructure, such as antennas, edges of a physical utility network, or even licenses with multi-years subscription commitments: redesigning the set of items is not an option, and each item has usage and maintenance costs that must be covered (even closing or neutralizing an item can be costly).[1]

An instance of our general model involves a fixed, finite, set R of useful "items", a specific cost for each item, and a set N of users/agents who must share the total cost of R. Agent i's needs are described by a set $\mathcal{D}^i$ of subsets S^i of R, dubbed *service sets*: agent i is served if and only if at least one of his service sets is provided. For instance in connectivity games where the items are the edges of the network, the set of paths between two nodes serves an agent who needs a single connection between these very nodes. Naturally $\mathcal{D}^i$ is inclusion monotonic, and hence entirely described by its subset $\bar{\mathcal{D}}^i$ of i's *minimal serving sets*, i.e., those serving sets from which no item can be removed without service interruption.

We propose and axiomatize a family of cost sharing methods, each one based on a *liability index*, $\theta(\mathcal{D}^i, a)$, capturing the importance of item a to agent i purely in terms of service options, independently of its cost. Then we simply divide the cost of each item in proportion to the profile of liability indices. The canonical liability index θ^1 in our family, that we call the *counting index*, is simply the ratio of the number of i's minimal serving sets containing a, to the total number of minimal serving sets. Formally, $\bar{\mathcal{D}}^i(a) = \{S \in \bar{\mathcal{D}}^i | a \in S\}$ and $\theta^1(\mathcal{D}^i, a) = \frac{|\bar{\mathcal{D}}^i(a)|}{|\bar{\mathcal{D}}^i|}$.

As our first example consider the elementary case where items have no substitutability for any agent: for all i, $\bar{\mathcal{D}}^i$ contains a single set S^i, meaning that agent i needs all items of S^i and nothing more.[2] We call the items in S^i *critical* for agent i, and we have $\theta^1(\mathcal{D}^i, a) = 1$ if a is critical, $= 0$ otherwise. Hence we divide the cost of each item equally between those agents for whom it is critical, or equally among all agents if this item is completely useless (is outside $\bigcup_i S^i$). This is also what the Shapley value of the corresponding Stand Alone cooperative game recommends.

Our second example involves full substitutability of items within a given set S^i, in the sense that any item in S^i is enough to meet i's needs, $\bar{\mathcal{D}}^i = \{\{a\} | a \in S^i\}$. Then $\theta^1(\mathcal{D}^i, a) = \frac{1}{|S^i|}$, which says that an agent with 10 equally good options has a liability of $\frac{1}{10}$ for each option. The corresponding division of costs is very simple, in sharp contrast with the Shapley value of the Stand Alone game that requires solving numerous combinatorial optimization problems.

Because liability indices determine the cost shares by simple proportionality, it is equally easy to mix the two examples above by assuming $\bar{\mathcal{D}}^i = \{S^i\}$ for some i while $\bar{\mathcal{D}}^j = \{\{a\} | a \in S^j\}$ for some j. Here is a very simple instance with three agents $1, 2, 3$ and two items a, b.

Agent 1 needs a, but has no use for b; agent 2 is served by either one of a and b, while agent 3 needs both a and b. Here, using our counting index, the cost of a is shared in proportion to $1, \frac{1}{2}, 1$ respectively, while the shares for b are $0, \frac{1}{2}, 1$. Compare this with the Stand Alone core, requiring agent 2 to pay at most the cost of the cheapest item. Combining this constraint with cost additivity forces 2 to pay nothing at all, as recommended in Moulin and Laigret (2011): letting agent 2 "free ride" is clearly unpalatable from the point of view of fairness. On the other hand the Shapley value of the Stand Alone game is not additive in costs; assuming a is cheaper than b, it splits three ways the cost of a, and charges the full cost of b to agent 3. Agent 2 is not liable at all for b, which would be helpful to her if a was unavailable: again an extreme interpretation of a fair division.

Our main result in Section 6 axiomatizes a one-dimensional family of cost ratios based on liability indices, including the counting index just discussed, and the egalitarian index of 1 for all items in the union of minimal serving sets, $\bigcup_{\bar{\mathcal{D}}^i} S^i$, and 0 outside this set. Besides the standard horizontal equity axioms with respect to agents and to items (Anonymity and Neutrality), the result uses three familiar axioms, and one new.

Consistency says that removing an agent i and reducing costs of each item by i's share does not affect the division among the remaining agents. It is arguably the most popular axiomatic requirement in the entire literature on the fair allocation of resources and costs (see e.g., Thomson, 2011).

Replication fixes the population N and considers two disjoint copies of the same pattern of needs, such that each agent needs service in both copies; then it must be the same to deal with the two problems separately or as a single big problem.

Additivity with respect to costs is a familiar restriction on cost allocation rules, going back to Shapley's original axiomatization (Shapley, 1953), and maintained throughout most of the axiomatic cost sharing literature, see e.g., Moulin (2002). We adopt it for the usual reasons of computational simplicity and cost decentralization.

The new property is the axiom *Irrelevance of Supplementary Items* (ISI), stating that if a new item appears, which creates no new minimal service options for any agent, then the liability index (and thereby the cost allocation) of the original set of items remains unchanged. ISI is very natural, and has much bite. It allows for instance to "merge" two items that always appear together in the minimal serving sets of all agents (we call these items *complements*): the single merged item inherits the common liability index of the two previous items.

[1] See Moulin and Laigret (2011) for more discussion of this assumption, and Moulin (2013) relating our model to the previous literature on cost sharing in networks.

[2] The classic *airport game* (Littlechild and Thompson, 1977) is an example.