Games and Economic Behavior 87 (2014) 508–518

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

A natural mechanism for eliciting rankings when jurors have favorites

Tsuyoshi Adachi [1]

Faculty of Political Science and Economics, Waseda University, 1-6-1 Nishiwaseda Shinjuku-ku, Tokyo 169-8050, Japan

ARTICLE INFO

Article history:
Received 31 March 2013
Available online 7 August 2014

JEL classification:
C72
D71
D78

Keywords:
Implementation theory
Backward induction
Subgame perfect equilibrium

ABSTRACT

We analyze the problem of a jury that must rank a set of contestants whose socially optimal ranking is common knowledge among jurors who may have friends among the contestants and may, therefore, be biased in their friends' favor. We show a natural mechanism that is finite and complete informational, with no simultaneous moves (i.e., it is solvable by backward induction), which implements the socially optimal ranking with subgame perfect equilibria.

1. Introduction

This paper considers the problem of a jury that must decide the top z ranks of $n(\geq z)$ contestants when the jurors may be biased. The socially optimal ranking has already been determined and is common knowledge among all jurors, but not verifiable. We can expect that the jurors are impartial in their evaluation of contestants they have no special interest in. However, each juror may have a stake in several contestants (e.g., some of them may be the juror's friends). As a result, the juror may prefer a different ranking than the socially optimal one. An example of this problem is a committee composed of professors that must rank students applying for scholarships. We can presume that professors know the true ranks but may favor their own students in the committee process. Examples of this problem can also be found in gymnastics competitions or competitive shows of foods, crafts, art, and so on. This paper examines the conditions where a jury can determine the true socially optimal ranking and considers related collective decision-making methods, based on a mechanism design approach.

Amorós et al. (2002) conducted the first study on this problem. They consider this problem in a framework where each juror is assumed to be biased in favor of a unique and distinct contestant and show that the socially optimal ranking is Nash implementable. Amorós (2009) considers a framework with a wider class of preferences—for example, a juror with one or more "friends" (contestants the juror would want to benefit) and "enemies" (contestants the juror would want to prejudice). Given this wide range, he provides the necessary and sufficient condition for the jury to determine the socially optimal ranking with respect to several equilibrium concepts, using decision-making methods similar to the canonical mechanism in Maskin (1999).

E-mail address: adachi39@gmail.com.

[1] Current address: Faculty of Economics, Takasaki City University of Economics, 1300 Kaminamie, Takasaki, Gunma 370-0801, Japan.

http://dx.doi.org/10.1016/j.geb.2014.07.003
0899-8256/

However, in the context of mechanism design theory, the canonical mechanism and other similar mechanisms have been criticized for the unnatural structures and the complexity of equilibrium strategies.[2] In this regard, Amorós (2011) studies implementation possibilities with a simpler mechanism by restricting the framework to a scenario in which each juror has a unique and distinct favorite contestant and the jury decides only the top rank (i.e., $z = 1$). He shows a natural extensive-form mechanism where the jurors decide, one by one, who would be the top contestant, leading ultimately to the socially optimal ranking with subgame perfect equilibria. Theoretically, this mechanism has a finite strategy space, and jurors have perfect information and make no simultaneous moves; thus, the subgame perfect equilibria in this mechanism are calculated by backward induction. This means that the jurors can easily understand the equilibrium strategies.[3]

This paper examines the implementation in a natural mechanism framework where the jurors have *one or more* friends, and the ranking is an *arbitrary length*. That is, we consider a more natural mechanism than that of Amorós (2009) in a more general framework than that of Amorós (2011). More precisely, we consider a class of preferences that are *impartial except for friends*, where a juror may have friends (but no proper enemies) and may want to rank them higher than they truly are. However, the juror would never want to do so for unfriendly contestants. We provide the necessary condition for implementation of a socially optimal ranking; for each pair of contestants, there exists at least one juror who would certainly not be friendly with both contestants. In addition, we show that this is also a sufficient condition for implementation with subgame perfect equilibria; thus, this is a necessary and sufficient condition. Further, we provide a *natural* mechanism that implements the socially optimal ranking.

In this mechanism, the jury sequentially decides the ranking from top to bottom, selecting the most appropriate contestants, among those not ranked, for each position. Specifically, for each unranked contestant, the jury gathers jurors who are certainly not friends with the contestant, and the jurors, with equal rights to make a decision, sequentially vote on whether the contestant is appropriate for the position. The strategy space in this mechanism is also finite, and the jurors have perfect information and make no simultaneous move; thus, this mechanism can be easily solved by backward induction.

The paper is organized as follows. In Section 2, we define a problem in which a jury decides ranking introducing a class of preferences that are impartial except for friends. We also define implementation of a socially optimal ranking, and give the necessary condition for its implementation. In Section 3, we describe the mechanism that implements the socially optimal rankings with subgame perfect equilibria. Finally, we discuss some impartiality conditions, present a few variants of our mechanism, and interpret our ranking problem in Section 4.

2. Setup

Let N be a set of n contestants, where $n \geq 2$. For each positive integer $l \leq n$, a sequence $\pi = (\pi_1, \pi_2, \ldots, \pi_l)$ of contestants is a *ranking* (with length l) if for each $m, m' \in \{1, 2, \ldots, l\}$, $\pi_m = \pi_{m'}$ implies $m = m'$. Let $\Pi(l)$ be the set of all rankings with length l. The jury J is a set of jurors who must decide the final ranking with a prospectively defined length z, with $1 \leq z \leq n$. Let $\Pi = \{\emptyset\} \cup [\bigcup_{l=1}^{z} \Pi(l)]$, where $\emptyset$ means no contestant is ranked. The socially optimal ranking in $\Pi(z)$ is common knowledge among jurors in J.[4] However, it is not verifiable. For each $\pi \in \Pi$ and each $a \in N$, p_a^π denotes the position of a in the ranking π; that is,

$$p_a^\pi = \begin{cases} m & \text{if } \pi_m = a, \\ l+1 & \text{if } a \notin \bigcup \pi_i, \end{cases}$$

where l is the length of π. Here, the second line of the formulation shows that all unranked contestants are, for convenience, considered identically positioned, posterior to ranked contestants.

Each juror $j \in J$ has a preference relation (i.e., a complete and transitive pairwise relation) R_j on the set of final rankings, namely $\Pi(z)$. Let $\mathcal{R}_j$ be the set of all preference relations and $\mathcal{R} = \prod_J \mathcal{R}_j$. We assume that for each juror j, N can be divided into two groups—a group composed of contestants with friendly relationships to j and a group composed of contestants in whom the juror has no stake—and each juror j is *impartial expect for friends* in the following sense: j prefers a modified ranking to a ranking where a contestant who is not j's friend is unjustly ranked highly as compared with the socially optimal ranking. Let $I_j \subseteq N$ be an *impartial set* of j: the set of the contestants in whom j has no stake. Thus, any contestant in I_j is not j's friend. The following is the formal definition:

Definition 1 *(Impartiality except for friends).* Suppose that j's impartial set is I_j and the socially optimal ranking is π^T. Then, $R_j \in \mathcal{R}_j$ is *impartial except for friends with respect to I_j and π^T* iff for each $(a, b) \in N \times I_j$ with $p_a^{\pi^T} < p_b^{\pi^T}$ and each $\pi, \pi' \in \Pi(z)$, such that

[2] See, for example, Jackson (1992); Jackson et al. (1994); Sjostrom (1994).

[3] Glazer and Perry (1996) also state these properties as reasons that their mechanism seems to be intuitive and simple.

[4] All results (including those in Appendices A and B.) hold even when the commonly-known socially optimal order on contestants is longer than z—the length of the ranking that the jury must decide—by defining *impartiality* at Definition 1 in accordance with the longer order, while assuming that its length is equal to z for simplicity.