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Lottery versus all-pay auction contests: A revenue dominance theorem ☆

Jörg Franke [a], Christian Kanzow [b], Wolfgang Leininger [a], Alexandra Schwartz [b]

[a] University of Dortmund (TU), Department of Economics, Vogelpothsweg 87, 44227 Dortmund, Germany
[b] University of Würzburg, Institute of Mathematics, Emil-Fischer-Str. 30, 97074 Würzburg, Germany

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ABSTRACT

We allow a contest organizer to bias a contest in a discriminatory way; i.e., she can favor specific contestants by designing the contest rule in order to maximize total equilibrium effort (resp. revenue). The two predominant contest regimes are considered, all-pay auctions and lottery contests. For all-pay auctions the optimal bias is derived in closed form: It implies extreme competitive pressure among active contestants and low endogenous participation rates. Moreover, the exclusion principle advanced by Baye et al. (1993) becomes obsolete in this case. In contrast, the optimally biased lottery induces a higher number of actively participating contestants due to softer competition. Our main result regarding total revenue comparison under the optimal biases reveals that the all-pay auction revenue-dominates the lottery contest for all levels of heterogeneity among contestants. The incentive effect due to a strongly discriminating contest rule (all-pay auction) dominates the participation effect due to a weakly discriminating contest rule (lottery).

1. Introduction

The all-pay auction and the lottery contest game are the most frequently used setups to model strategic competition among agents that exert non-refundable effort to influence their respective chances to win a fixed prize. Both types of models have the advantage that they are simple and easy to understand. Moreover, they are familiar from real world applications, for instance, in the areas of R&D competition, lobbying, sports, rent-seeking, procurement, etc., see Konrad (2009) for a survey. Besides being the first setups that historically emerged in the economic analysis of these applications, their use can also be justified on the grounds of a number of axiomatizations and micro-foundations, see the mentioned survey for references and, in particular, Clark and Riis (1998) for unfair contests. Another important reason for the popularity of these models is their analytical tractability, especially if employed under the assumptions that the rules that govern the competition are anonymous and that agents are homogeneous. Recently, there is a growing interest in relaxing these limiting assumptions: The heterogeneity of contestants becomes the focus of analysis and, as a consequence, also the question of how the contest organizer should exploit the heterogeneity among contestants by treating different contestants differently. Recent examples that follow this approach are Siegel (forthcoming, 2010), Kirkegaard (2012), Epstein et al. (2011), Szech

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E-mail addresses: Joerg.Franke@tu-dortmund.de (J. Franke), kanzow@mathematik.uni-wuerzburg.de (C. Kanzow), Wolfgang.Leininger@tu-dortmund.de (W. Leininger), schwartz@mathematik.uni-wuerzburg.de (A. Schwartz).

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(2011), Fu (2006), Harbaugh and Ridlon (2011), Franke (2012), and Franke et al. (2013). A mechanism design perspective on contests involving handicaps and heterogeneous contestants is used in Che and Gale (2003).

Due to the prominence of the rent-seeking interpretation in this literature an important aspect in the strategic analysis is the relation between aggregate equilibrium efforts of the agents (i.e., the revenue of the auction or contest) and the underlying institutional rules and characteristics that govern the specific form of the contest. In Baye et al. (1993), for instance, an analysis of the all-pay auction with heterogeneous players established the so-called exclusion principle, which implies that a revenue-maximizing contest organizer might optimally exclude strong agents from the competition *ex ante*. This result is in contrast to the symmetric lottery case considered in Fang (2002), where it is shown that exclusion of strong players is never optimal for the contest organizer. Moreover, the direct comparison between these two contest regimes reveals that neither revenue-dominates the other a priori. The intuition for this result can be attributed to the trade-off between competitive pressure and active participation of contestants which is differently resolved in the two regimes: Competitive pressure in an all-pay auction is primarily generated by the institution itself. Its highly discriminative, all-pay deterministic winner-takes-it-all nature implies that in equilibrium (generically) only two contestants will endogenously decide to actively participate in the auction. However, competition between those two is so intense, that in (mixed-strategy) equilibrium only one contestant has a positive payoff in expectation (while both have a positive probability of winning). In contrast, a lottery contest with its characteristic probabilistic contest rule is much less discriminative as an institution because it does not require to be the highest bidder in order to win. This characteristic is highly conducive for attracting active participation by contestants; i.e., competitive pressure in a lottery contest is primarily generated by the interaction of many active contestants in equilibrium (see Fu and Lu, 2010, for an analysis of the relation between contest revenue and endogenous entry of homogeneous contestants depending on entry fees and subsidies). Fang (2002) shows that from a revenue-maximizing contest designer's point of view it depends on the degree and nature of heterogeneity of contestants whether it is better to ignite competitive pressure *ex ante* (through the choice of a very discriminative contest success function like the all-pay auction), which is reduced endogenously *ex post* due to a minimal number of actively participating contestants, or to opt for weaker competitive pressure *ex ante* (by choosing a less discriminative contest success function like the lottery contest) which is endogenously reinforced *ex post* due to a large number of actively participating contestants.

Importantly, both of these models are based on the assumption that the contest organizer is neutral with respect to the contestants; that is, she chooses among contest regimes, which treat contestants anonymously. This is certainly not the case in many real world contests (just think of the contest rules for a job opening of a professorship), where the contest organizer has control over some variables, which bias the contest systematically (and legally) in favor of certain contestants. Further examples are provided by biased contests in affirmative action contexts, public procurement practices, which favor local or national firms over others, sport tournaments with handicap schemes, and litigation law, which allocates the costs between the parties involved asymmetrically. A detailed account of biased public procurement in Israel is provided by Epstein et al. (2011). Biasing the contest rule gives the contest organizer additional power to promote her interests, in particular in the presence of heterogeneous contestants. This situation is analyzed for the case of two contestants in Epstein et al. (2011), where the contest organizer can specify individual weights for each of two contestants. Setting individual weights reflects her potential for discriminating between the two contestants which has consequences for the revenue comparison between all-pay auction and lottery contest: The optimally biased all-pay auction revenue-dominates any biased lottery contest, independently of heterogeneity between the two contestants. However, the restriction to the two-player case is particularly severe for at least two reasons: Firstly, it ignores the basic trade-off with regard to competitive pressure as described above. The "minimal participation" feature of the all-pay auction is eliminated, likewise the scope of the lottery contests for increased competitive pressure through additional active contestants. Secondly, the solution theory of the biased lottery contest with only two contestants is a degenerate case of the general n-player solution, see Franke et al. (2013). More precisely, the optimal weight for a contestant in the two-player case only depends on his own characteristics, whereas with three or more players any optimal individual bias weight depends on the characteristics of *all* contestants.

The objective of this paper is to determine a revenue (or total effort) maximizing contest organizer's choice of contest, when she is faced with n heterogeneous contestants. Her choice set consists of a set of (potentially biased) contest success functions, which contains lotteries and all-pay auctions. For lottery contests we can rely on Franke et al. (2013), who analyze the optimal choice of the contest organizer if her choice set is restricted to biased lotteries. However, the optimal choice from the set of biased all-pay auctions has not been determined so far. This derivation is challenging due to the fact that, depending on the choice of the all-pay auction contest rule, multiple mixed-strategy equilibria might exist which are not revenue equivalent. Nevertheless, we derive a simple expression of the optimal bias in closed form and the corresponding revenue for any finite set of contestants with heterogeneous valuations. This result allows us then to compare the induced revenue in the two regimes under the respective optimal biases. Our second main result (Theorem 4.3) states that revenue dominance of an optimally biased all-pay auction over the optimally biased lottery holds for any given set of heterogeneous contestants. This result is far from trivial, but has a clear intuition: The ability of the contest organizer to discriminate between contestants in the all-pay auction is used to make the exclusion principle obsolete (an alternative approach is the modified all-pay auction rule in Gale and Stegeman, 1994, which gives only the strongest contestant a special status). Under the optimal bias it will always be the two strongest contestants who choose to be active, and they are made to compete with each other in the strongest possible way, i.e., in a playing field that is completely leveled due to the bias. No strong player is excluded a priori by the organizer. As expected, the discriminatory power of the contest organizer in the lottery contest is used to encourage more contestants to actively participate: In any optimally biased lottery contest