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Universal type structures with unawareness

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ABSTRACT

Infinite hierarchies of awareness and beliefs arise in games with unawareness, similarly to belief hierarchies in standard games. A natural question is whether each hierarchy describes the player's awareness of the hierarchies of other players and beliefs over these, or whether the reasoning can continue indefinitely. This paper constructs the universal type structure with unawareness where each type has an awareness level and a belief over types. Countable hierarchies are therefore sufficient to describe all uncertainty in games with unawareness.

1. Introduction

There are many situations where the payoff of an agent depends on the actions of other agents and on uncertain external factors. The actions of the agents depend on their reasoning, so to choose the best action, an agent has to reason about the reasoning of others, their reasoning about the reasoning of others, and so on to arbitrarily high order. With complete information, this infinite regress can be avoided by imposing a fixed-point equilibrium concept, but with incomplete information, the process generates infinite hierarchies of reasoning. A natural question is whether the infinite hierarchies summarize all the uncertainty in the game or whether it is necessary to go even further, describing a player's reasoning about the opponents' infinite hierarchies, about their reasoning about their opponents' hierarchies, etc.

In standard games, the reasoning of an agent is described by a probability distribution over known outcomes (exogenous uncertainties and other agents' possible beliefs). The seminal paper of Mertens and Zamir (1985) was the first to show that the initial hierarchies of reasoning are sufficient—each hierarchy encodes a probability distribution over the set of hierarchies, thus closing the model.

Not all uncertainty is describable by a probability distribution, but infinite regress is a feature of any kind of interactive reasoning. The same question of closure of the hierarchical model then arises as for probabilistic beliefs. If the uncertainty is described by conditional probability systems, compact continuous possibility correspondences or compact sets of probabilities, the papers of Battigalli and Siniscalchi (1999), Mariotti et al. (2005) and Ahn (2007) show that the hierarchies capture all uncertainty about the hierarchies.

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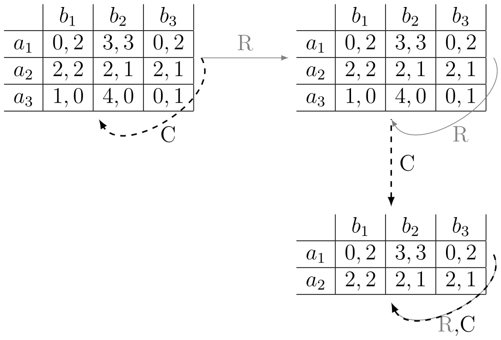

Fig. 1. A game with unawareness from Feinberg (2009).

On the other hand, for knowledge and similar information structures, reasoning about other agents' reasoning may continue indefinitely, as shown in Heifetz and Samet (1998a), Meier (2005), therefore there may be no level of the hierarchies that fully describes all uncertainty in the model. With additional assumptions on the knowledge operators, closure can be obtained (Meier, 2008; Mariotti et al., 2005).

All the above mentioned kinds of uncertainty share the property that the agents know all possible outcomes. In many real-world decision problems this is not the case—the agents may be unaware of some aspects of the environment. An easily noticeable difference between unawareness and zero probability is that unawareness is symmetric—if an agent is unaware of an event, he is unaware of its negation. Zero probability on an event implies putting probability one on its negation. Choice under unawareness may exhibit 'reverse Bayesianism', i.e. updating may lead an initially null event to receive positive weight in decisions (Karni and Vierø, 2010). The experiment of Mengel et al. (2011) shows that being exposed to unawareness increases the risk aversion of experimental subjects.

Unawareness may give rise to novel behavior, as illustrated in the game in Fig. 1, which consists of three normal form games, with beliefs (represented by arrows) between them. At the top left normal form, the column player C believes the game to be the full Fig. 1, while R believes the game to be restricted to the two normal forms on the right side. At the top right normal form, R again believes the game to be restricted to the two normal forms on the right side, while C believes the game to consist of just the bottom right normal form (a standard game). At the bottom right normal form, both believe the game to be the bottom right. So at the top left game, C believes that R believes that C believes the bottom right normal form is the whole game. All beliefs put probability one on a single game in this example, but this need not be the case in other games with unawareness.

The solution of the game in Fig. 1 is reminiscent of backward induction. In the bottom right normal form, there are two pure-strategy equilibria: (a_2, b_1) and (a_1, b_2). Focus on the payoff-dominant (a_1, b_2). In the game consisting of the two right-side normal forms, R believes C imagines himself in the bottom right normal form and plays b_2. Then R best responds with a_3. In the whole game, C believes R imagines herself to be in the game consisting of the two right-side normal forms and plays a_3, so C best responds with b_3. The equilibrium (a_3, b_3) of the game with unawareness is absent from both the standard game consisting of just the top left normal form in Fig. 1 and from the standard game consisting of just the bottom right normal form. The addition of unawareness has changed the equilibrium set.

To better describe such situations, unawareness has been added to games by Grant and Quiggin (2009), Halpern (2008), Rêgo and Halpern (2012), Heifetz et al. (2011b). In games with unawareness, the players are aware of only some aspects of the game, form beliefs about external uncertainties and other players' awareness and beliefs, and so on, giving rise to infinite hierarchies of awareness and belief. Similarly to standard games, there is a question as to whether the hierarchies encapsulate all uncertainty in the model. Given the results in the literature that the hierarchical model closes for some kinds of uncertainty, but not for others, the answer is not obvious in the case of unawareness.

This paper proves that infinite hierarchies of awareness and belief include the description of awareness and belief about the hierarchies if agents are unaware of aspects of the space of primitive uncertainty, other agents or higher-order reasoning.

In propositional unawareness, the space of primitive uncertainty is the set of states of nature. Awareness levels are based on partitions of the space of primitive uncertainty. An agent with a particular awareness level can reason about the partition corresponding to that level, about others' reasoning about that partition, their reasoning about his reasoning about the partition, etc. Propositional unawareness is the most common kind used in the literature, in both propositional models (Halpern, 2001; Heifetz et al., 2008; Halpern and Rêgo, 2008) and set-based ones (Modica and Rustichini, 1999; Heifetz et al., 2006; Li, 2009). Unawareness of available actions, as in Feinberg's game in Fig. 1 and in Heifetz et al. (2007, 2011a), can be described using the same mathematical framework as for propositional unawareness.[1] This is made possible by using the space of primitive uncertainty to describe uncertainty about the availability of actions, as well as payoff uncertainty. It

[1] The author thanks an anonymous referee for this suggestion.