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On the continuous equilibria of affiliated-value, all-pay auctions with private budget constraints ☆

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ABSTRACT

We consider all-pay auctions in the presence of interdependent, affiliated valuations and private budget constraints. For the sealed-bid, all-pay auction we characterize a symmetric equilibrium in continuous strategies for the case of N bidders. Budget constraints encourage more aggressive bidding among participants with large endowments and intermediate valuations. We extend our results to the war of attrition where we show that budget constraints lead to a uniform amplification of equilibrium bids among bidders with sufficient endowments. An example shows that with both interdependent valuations and private budget constraints, a revenue ranking between the two auction formats is generally not possible. Equilibria with discontinuous bidding strategies are discussed.

Suppose firms are lobbying for a lucrative government contract. The contract's value to each firm has an idiosyncratic component since the firms likely have different operating costs. But also, each firm has a privately-known limit on how much it is *able* to spend on the lobbying game. Perhaps the management of one firm is approving of restaurant meals with officials but expenditures or bribes beyond some threshold are morally too much to stomach. A competitor, in contrast, may be less hampered in its lobbying strategy. How does the lobbying game unfold when competitors differ in their valuation for the prize and in their ability or capacity to compete for it? Would some firms spend *more* on lobbying believing that their competitors have to navigate within some private and binding constraints on actions?

In this essay we consider a class of situations not unlike the above lobbying contest by analyzing all-pay auctions. In a (first-price) all-pay auction, the highest bidder is the winner of the item for sale; however, all bidders incur a payment equal to their bid. As a stylized model of a lobbying contest, the all-pay auction has an established tradition in political economy (Hillman and Riley, 1989; Baye et al., 1993).[1]

Despite the frequent application of the all-pay auction to models of contests, most analyses fail to capture the exogenous, but private, limits on actions that are commonly encountered. In practice all participants face a budget constraint, a hard

☆ This paper generalizes our previous working paper, "All-Pay Auctions with Budget Constraints: The Two-Bidder Case." That paper merged the independent and simultaneous work of Kotowski (2010) and Li (2010). We are grateful to the editor and the referees for helpful comments.

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[1] We focus only on auction mechanisms where a bidder placing the highest bid is the winner. Probabilistic contests in the sense of Tullock (1980) are beyond this paper's scope. When we refer to "contests" we have in mind the special case that we are analyzing. Konrad (2009) provides a survey of the literature on contests more broadly, and includes a discussion of all-pay auctions.

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deadline, or a maximum level of feasible effort. Ignoring these constraints has hitherto been a helpful modeling simplification. We argue, however, that this simplification has masked much of the nuance embedded in the situation. Our analysis introduces private constraints into the all-pay auction with interdependent and affiliated valuations. We identify sufficient conditions for the existence of an equilibrium in continuous strategies. We also provide an extension of our model to the (static) war of attrition to show the broader applicability of our analysis.

Although our model is phrased in the language of auctions (players are called "bidders," etc.), it applies to any situation where resources are irreversibly expended in pursuit of a goal or a prize. The goal or prize can have a value that has both private and common components. Our model accommodates both cases. The private constraints on bids or effort that we introduce are often natural elements of the situation. The constraints may be financial, physical, or amalgams of many component factors.

As one specific example of the range of applications, consider college admissions. Hickman (2011) employs a version of the all-pay auction to model students competing for places in a college. Those who "bid" the most, by exerting irreversible effort, are more likely to gain a scarce spot in the school. It is clear that the benefit from a college degree varies across students due to personal preferences and characteristics. Hence, different students value college attendance to varying degrees. It is also natural to assume that idiosyncratic shocks, such as health status, family background, parental savvy, or school location, place an exogenous, heterogeneous, and private cap on the effort that a particular student can exert in the college admissions game.[2]

As another example, consider a patent race between competing firms. Such competition is naturally modeled as either an all-pay auction or as a war of attrition (Leininger, 1991). The expected value of the invention and the budget available to a company's research division will determine the effort devoted to the race. Information asymmetries or agency concerns can create a wedge between the available budget and the research division's assessment of the project's value. Moreover, each firm likely faces a hard, short-run physical resource constraint. This constraint will cap its feasible effort level. The interaction between expected rewards and heterogeneous resource constraints will shape how firms engage in this competition.

While we are motivated by the range of social and economic situations that all-pay auctions can model, our study also fills a gap in the growing literature on auctions with private budget constraints. Our analysis builds directly on the work of Krishna and Morgan (1997) who study the all-pay auction and the war of attrition with interdependent and affiliated valuations. Their analysis extends the general symmetric model of Milgrom and Weber (1982) to these more unusual auction procedures. To this setting we introduce private budget constraints distributed continuously on an interval. Our environment parallels the setting of Fang and Parreiras (2002, 2003) and Kotowski (2013) who study the second-price and the first-price auction with private budget constraints, respectively. These latter studies build directly on Che and Gale (1998b), which is the seminal paper in the literature on standard auctions incorporating private budget constraints. Che and Gale (1996) develop a simple model of an all-pay auction with private budget constraints where the item for purchase has a common and perfectly known value. That model is a limiting case of our environment. Finally, there is also a literature on publicly-known spending or bidding caps in all-pay auctions, or in contests more generally (Che and Gale, 1998a; Gavious et al., 2002). In our study, the spending or bidding limit of each bidder is *private* information.

In light of this literature, our study contributes along several dimensions. First, by focusing on all-pay mechanisms we put under scrutiny an important allocation mechanism in resource-constrained environments. Many authors examining optimal auctions with budget-constrained participants have resorted to mechanisms that feature "all-pay" payment schemes (Laffont and Robert, 1996; Maskin, 2000; Pai and Vohr, 2014). Our analysis therefore complements this literature, but we do not attempt the mechanism design exercise here.

Second, our model is set in a more general environment than traditionally employed when analyzing auctions with private budget constraints. Hence, we are able to identify additional features of the environment that affect the existence of a well-behaved and (relatively) tractable equilibrium. Previous studies lodged in the affiliated and interdependent-value paradigm, such as Fang and Parreiras (2002) and Kotowski (2013), have focused on the two-bidder case. While some of the intuition from the two-bidder case is relevant generally, the case of two bidders masks many caveats. For example, in the all-pay auction we document how changes in the number of bidders alone directly affect the existence of an equilibrium within the class of strategies traditionally considered by this literature. This observation may be particularly valuable to future empirical analyses as it may be difficult to exploit a variation in the number of bidders to aid in model identification (Athey and Haile, 2007).

The equilibria that we construct in the all-pay auction and in the war of attrition are in monotone, continuous strategies. As discussed by Araujo et al. (2008), non-monotone equilibria often feature in multi-dimensional auction environments.[3] In our setting, bidders have two dimensions of private information—a value-signal and a budget constraint—and interdependent valuations. Therefore the issues they address are related to our analysis.[4] We view our focus on monotone equilibria in continuous strategies as a pragmatic but reasonable choice. Although monotonicity is not a necessary condition to leverage the differential approach when characterizing equilibrium bidding (Araujo et al., 2008), it greatly simplifies our argument.

[2] For a discussion concerning the strategic aspects of college applications and admissions see Avery et al. (2004).

[3] See also Zheng (2001).

[4] Araujo et al. (2008) also propose an interesting application of the all-pay auction as a tie-breaking device in more complex auction-like games. Except for the analysis of Section 4, ties do not occur in our model.