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Beliefs and rationalizability in games with complementarities

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ABSTRACT

We propose two characteristics of beliefs and study their role in shaping the set of rationalizable strategy profiles in games with incomplete information. The first characteristic, *type-sensitivity*, is related to how informative a player thinks his type is. The second characteristic, *optimism*, is related to how "favorable" a player expects the outcome of the game to be. The paper has two main results: the first result provides an upper bound on the *size* of the set of rationalizable strategy profiles; the second gives a lower bound on the change of *location* of this set. These bounds are explicit expressions that involve type-sensitivity, optimism, and payoff characteristics. Our results generalize and clarify the well-known uniqueness result of global games (Carlsson and van Damme, 1993). They also imply new uniqueness results and allow us to study rationalizability in new environments. We provide applications to supermodular mechanism design (Mathevet, 2010b) and information processing errors.

1. Introduction

In all social or economic interactions, agents' beliefs contribute to shaping the set of outcomes. In game-theoretical models, the richness of outcomes is captured by the set of rationalizable strategy profiles. The global game literature (e.g., Carlsson and van Damme, 1993; Frankel et al., 2003 and Morris and Shin, 2003) suggests a perturbation of complete information that delivers a unique rationalizable equilibrium. This perturbation gives players' beliefs the right properties to obtain uniqueness. What are these properties? How do they interact with the payoffs to determine the rationalizable outcome? The standard global game method does not cover games with non-common prior type spaces, games played by individuals with updating biases, and Bayesian mechanism design. In these cases, our understanding of rationalizability requires an answer to the above questions.

In this paper, we study properties of type spaces that explain the size and the location of the set of rationalizable strategy profiles, where rationalizability corresponds to the concept of interim correlated rationalizability by Dekel et al. (2007). Type spaces provide the framework to model incomplete information. In our formulation, there is a state of nature, and each player has type-dependent beliefs about the state of nature and others' types. We study properties of type spaces in games with complementarities in which players care only about an aggregate of their opponents' actions, such as their

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average action. We also assume that there exist dominance regions, that is, "tail regions" of the state space for which the extremal actions are strictly dominant. The model applies to many classic problems such as investment games, currency crisis, and search models.

The analysis does not require that we specify the origin of the beliefs. As in Van Zandt and Vives (2007), our formulation of the Bayesian game is interim. Since our main conditions are defined on the interim beliefs, they are compatible with general belief formation, including heterogeneous beliefs and information processing errors.

The first characteristic that we study is type-sensitivity of the beliefs. This notion has two dimensions, one for the beliefs about the state and one for the beliefs about others' types. The first dimension is the answer to the question: when a player's type increases, by how much does he think the state will increase on average? This question gives information about how informative the player thinks his type is. The second dimension applies to the beliefs about others' types. We want to know if a player believes that other players' types increase more than his when his own type increases. To find out, we ask the following question: assuming that players other than i decrease their strategies while i's type increases, by how much will the aggregate action decrease on average according to i? The answer is the second dimension of type-sensitivity. Since the player's type increases, he believes others' types will increase as well. As a result, he may believe that his opponents will play larger actions although their strategies decrease, and so a larger aggregate may occur.

The second characteristic is optimism of players' beliefs. This characteristic has two dimensions and measures how favorable a player expects the outcome to be. By convention, the outcome is more favorable if the aggregate and the state are larger. Thus, a player becomes more optimistic if, at any type, he now believes larger states and larger aggregates are more likely than before. In technical terms, an increase of optimism is represented by a first-order stochastic dominance shift of the beliefs for each type.

Before stating our two main results, recall that there exist a largest and a smallest equilibrium in supermodular games. The distance between them gives the size of the set of rationalizable strategy profiles (Milgrom and Roberts, 1990).

The first result provides an explicit upper bound on the size of the set of rationalizable strategy profiles. The second result provides an explicit lower bound on the movement of the rationalizable strategy profiles after a change of optimism. Both bounds are explicit and simple expressions involving type-sensitivity, optimism, and characteristics of the payoffs. These expressions are easy to compute in comparison to applying iterative dominance and computing the rationalizable outcomes directly.

On the one hand, our results imply new uniqueness results and promote a better understanding of global games. The global game method suggests a specific perturbation of complete information that delivers a unique rationalizable equilibrium. But many scenarios do not fit into the global game description: games with heterogeneous beliefs (non-common prior type spaces), information processing errors in games, and Bayesian mechanism design. To study these cases, it is important to understand the properties of type spaces inherited from the global game perturbation. Type-sensitivity and optimism are such properties. The upper bound provided by the first result subsumes the global game uniqueness result and shows that uniqueness holds more generally than in global games. We illustrate this in Section 2 in an investment game. The bound also shows *explicitly* that the global game information structure dampens the complementarities to the point where a unique equilibrium survives. This formalizes arguments presented by Vives (2004) and Mathevet (2010a).

On the other hand, the results allow us to study equilibrium multiplicity. While the literature has focused on uniqueness, it is important to understand and quantify equilibrium multiplicity. In supermodular mechanism design, for example, knowing the size of the equilibrium set allows us to compute the welfare loss that may be caused by bounded rationality (Mathevet, 2010b). Our results show that larger type-sensitivity is conducive to tighter equilibrium sets. Furthermore, certain characteristics of equilibrium multiplicity are interesting. For example, as we move from one equilibrium to another, some players may change their equilibrium strategy more dramatically than others. It seems natural to say that a player whose equilibrium strategy varies less across equilibria is more influential in the game than one whose equilibrium strategy is more responsive. In Section 6.3, we identify the more influential players as those having higher type-sensitivity.

We apply our results to supermodular mechanism design (Mathevet, 2010b) and information processing errors. The idea behind supermodular mechanisms is to design mechanisms that induce games with strategic complementarities, because they are robust to certain forms of bounded rationality (Milgrom and Roberts, 1990). Although strategic complementarities are helpful, excessive complementarities may produce new equilibria and disrupt learning.[2] This justifies the concept of an optimally supermodular mechanism, one that gives the smallest equilibrium set among all supermodular mechanisms (Mathevet, 2010b, Theorem 3). But what is the size of the smallest equilibrium set? Our first result can provide an answer and help the designer choose the right parameter values in the mechanism. In Section 6, we also study games played by players with updating biases. Information processing errors can often be interpreted as heterogeneity in the players' priors (Brandenburger et al., 1992), hence type-sensitivity and optimism are useful. We argue that the underreaction bias favors multiplicity, while the overreaction bias favors uniqueness.

The importance of understanding rationalizability beyond global games is emphasized by Morris and Shin (2009). They characterize the hierarchies of beliefs that imply dominance-solvability in binary-action games with incomplete information. Our paper formulates alternative conditions in games with finitely many actions. We will discuss the relationship between type-sensitivity and their notion of decreasing rank beliefs. Izmalkov and Yildiz (2010) is another paper close to ours. The

[2] This is because Mathevet (2010b) studies weak implementation and truthtelling is the only equilibrium known to be desirable.