Games and Economic Behavior 85 (2014) 272–288

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Robustness to strategic uncertainty

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ARTICLE INFO

Article history:
Received 1 March 2012
Available online 26 February 2014

JEL classification:
C72
D43
L13

Keywords:
Nash equilibrium
Refinement
Strategic uncertainty
Bertrand competition
Log-concavity

ABSTRACT

We introduce a criterion for robustness to strategic uncertainty in games with continuum strategy sets. We model a player's uncertainty about another player's strategy as an atomless probability distribution over that player's strategy set. We call a strategy profile robust to strategic uncertainty if it is the limit, as uncertainty vanishes, of some sequence of strategy profiles in which every player's strategy is optimal under his or her uncertainty about the others. When payoff functions are continuous we show that our criterion is a refinement of Nash equilibrium and we also give sufficient conditions for existence of a robust strategy profile. In addition, we apply the criterion to Bertrand games with convex costs, a class of games with discontinuous payoff functions and a continuum of Nash equilibria. We show that it then selects a unique Nash equilibrium, in agreement with some recent experimental findings.

1. Introduction

In recent experimental studies, uncertainty about other players' actions has been found to be a clear driver of behavior, see e.g. Heinemann et al. (2009) and Cabrales et al. (2010). Following Harsanyi and Selten (1988) and Brandenburger (1996), one usually alludes to this type of uncertainty as 'strategic uncertainty', as opposed to uncertainty regarding the underlying structure of the game played, which is sometimes called 'structural uncertainty' (see e.g. Morris and Shin, 2002). Strategic uncertainty matters because in a wide range of games, many of the equilibria represent fragile situations in which players are supposed to choose a particular strategy, even though this is optimal only if they hold knife-edge beliefs about the actions taken by other players. In such situations, even the slightest uncertainty about other players' choices might lead a player to deviate from his or her equilibrium strategy. This uncertainty problem is aggravated in games with multiple Nash equilibria, and it may get particularly serious when there is a whole continuum of equilibria.

That said, in the laboratory, human subjects' behavior in games with multiple equilibria has also been found to be fairly stable and predictable. For instance, Abbink and Brandts (2008) produce an experimental study of Bertrand competition under strictly convex costs. Dastidar (1995) had shown that those oligopoly games admit a whole continuum of Nash equilibria, but they find that an attractor of play is the zero-monopoly-profit price. In experimental treatments with more than two firms in the market, that price is actually the modal outcome in their data. Abbink and Brandts (2008) remark that

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http://dx.doi.org/10.1016/j.geb.2014.01.018
0899-8256/

"[that] price level (...) is not predicted by any benchmark theory [they] are aware of" (p. 3).[1] We conjecture that part of this regularity may be that some equilibria are perceived as less strategically risky than others. In this paper, we introduce, and study in some generality, a robustness criterion for games with continuum strategy sets, a criterion which we argue is particularly operational. We then proceed to show that our robustness criterion selects a unique equilibrium in the game of Bertrand competition with convex costs, and that this selection agrees with Abbink's and Brandts' (2008) empirical findings.

To be more specific about our contribution, we here formalize a notion of strategic uncertainty and propose a criterion for robustness to such uncertainty. We focus on games with continuum action spaces, with measurable and bounded, but not necessarily continuous, payoff functions. Our approach is, roughly, as follows. A player's uncertainty about others' strategy choices is represented by a player-specific full-support atomless probability distribution, scaled with a parameter $t \geqslant 0$. For each value of the uncertainty parameter t, we define a t-*equilibrium* as a Nash equilibrium of the game in which each player strives to maximize her expected payoff under her strategic uncertainty so defined. For $t = 0$, this is nothing else than Nash equilibrium in the original game. We call a strategy profile *robust to strategic uncertainty* if there exists a collection of probability distributions in the admitted class, one for each player, such that some accompanying sequence of t-equilibria converges to this profile as the uncertainty parameter t tends to zero. If convergence holds for all distributions in the admitted class, we say that the strategy profile is *strictly robust* to strategic uncertainty (in this class). For games with continuous payoff functions, we show that our criterion is a refinement of Nash equilibrium and that under standard (compactness and convexity) assumptions, robust equilibria exist. While Nash equilibrium is the answer to the question "What strategy profiles are such that each player finds his strategy optimal if he is certain all others play theirs?" we thus here ask "What strategy profiles are such that each player has an optimal strategy near her strategy if she believes others are likely to play near theirs?"

Because it makes use of continuous distributions over the strategy sets, the proposed framework is well-suited to study games with discontinuous payoff functions, which are not uncommon in economics. For instance, when applied to Bertrand competition, strategic uncertainty results in uncertainty-perturbed profit functions that are continuous. In our view, the virtues of our approach therefore lie in its ease of use and its predictive power.

Those two characteristics also motivated the introduction of quantal response equilibrium (QRE) by McKelvey and Palfrey (1995). Although this approach was initially developed for finite games, Anderson et al. (1998) extended it to infinite-action games. The QRE approach assumes probabilistic choice, driven either by (unmodelled) idiosyncratic perturbations of individual players' preferences or by mistakes in their implementation of strategies. The choice probabilities are increasing functions of the expected payoffs. QRE requires consistency of beliefs in the sense that each player's probability distribution follows from its presumed functional form as applied to the expected payoffs from other players' equilibrium probabilistic choices. However, players do not best-respond to their information or beliefs about other players' actions; this occurs only in the limit as the noise level is driven down to zero. In a sense, our approach is symmetric to the one in QRE. Nash equilibrium requires beliefs to be consistent with others' actions, and actions to be optimal under those beliefs. QRE maintains consistency of beliefs with actions but relaxes the requirement that actions be optimal. Our notion of t-equilibrium instead relaxes the requirement that beliefs be consistent with others' actions but maintains the requirement that actions be optimal. In the limit, as the noise vanishes, both approaches will tend to agree (in distribution) with the assumptions of Nash equilibrium but will approach them from different directions. As we show in Section 3.1 this may lead to different predictions even in games with compact and convex strategy sets and continuous and concave payoff functions.

Baye and Morgan (2004) apply the infinite-action QRE to a class of Bertrand games that includes our main application. Assuming that the noise terms have identical power distributions, they show the existence of a symmetric limit QRE that agrees with the price that we show is robust to strategic uncertainty. By contrast, we do not assume symmetry. Instead, we derive both symmetry and uniqueness, and also show that the selected price is in fact strictly robust, that is, robust for a wide range of probabilistic beliefs about others' strategy choices.

We view our approach as a generalization to continuum-action set games of the concept of substitute-perfection introduced by Selten (1975) for finite games. Selten considers a perturbed world in which players play full-support mixed strategies. Substitute-perfection of a particular profile of strategies requires that each player continues to find it optimal to play the strategy prescribed by that profile when the perturbations are sufficiently small. We work in a similarly perturbed world but we do not require players to play exactly the strategy prescribed by a robust strategy profile. That is too strong a requirement for general payoff functions. (In the mixed-strategy extensions of finite games with which Selten works, payoff functions are necessarily multilinear.) Instead, we require players to choose a best response that is close to the one prescribed by the robust profile, for small enough perturbations.

We are not the first to extend perfection ideas to infinite games. Simon and Stinchcombe (1995), Méndez-Naya et al. (1995), and Bajoori et al. (2013) study perfection and related ideas in games with compact and convex strategy sets, and continuous payoff functions. Carbonell-Nicolau (2011a, 2011b) extends the notion of trembling-hand perfection to games with discontinuous payoff functions in a fashion that is shown to be equivalent to strong perfection in the sense of Simon and Stinchcombe (1995). Contrary to us, those authors work with mixed strategies whereas we work with subjective beliefs. To establish a connection with their approaches we define *interpersonally consistent* beliefs where any two players' beliefs

[1] Argenton and Müller (2012) corroborate this experimental finding in a laboratory experiment with other subjects and parameter values. van Huyck et al. (1990, 1991) (in the case of minimum effort games) and Heinemann et al. (2009) (in the case of coordination games) also identify rather predictable patterns of play in spite of the multiplicity of equilibria.