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Note

Ranking asymmetric auctions: Filling the gap between a distributional shift and stretch ☆

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ABSTRACT

I consider first-price auctions (FPA) and second-price auctions (SPA) with two asymmetric bidders. The FPA is known to be more profitable than the SPA if the strong bidder's distribution function is convex and the weak bidder's distribution is obtained by truncating or horizontally shifting the former. In this paper, I employ a new mechanism design result to show that the FPA remains optimal if the weak bidder's distribution falls between the two benchmarks in a natural way. The same conclusion holds if the strong bidder's distribution is concave, but with a vertical shift replacing the horizontal shift. A result with a similar flavor holds if the strong bidder's distribution is neither convex nor concave. The dispersive order and the star order prove useful in comparing the weak bidder's distribution to the benchmarks. A key step establishes a relationship between these orders and reverse hazard rate dominance.

1. Introduction

In the by now standard independent private values model, the celebrated *Revenue Equivalence Theorem* implies that the auction format is irrelevant for a risk-neutral seller whenever bidders are homogeneous ex ante. While Vickrey (1961) discovered an early version of the Revenue Equivalence Theorem, he also proved that it does not extend to a more realistic setting with heterogeneous bidders. Since then, a literature has emerged comparing the standard auctions, specifically the first-price auction (FPA) and the second-price auction (SPA). This literature has identified a number of isolated environments or examples, each of which allows a revenue ranking to be obtained. In this paper, I identify robust classes of environments where the FPA can be shown to be superior to the SPA.[1]

In a seminal paper, Maskin and Riley (2000) study three particular environments. In the first model, the strong bidder's distribution, F_s, is obtained by shifting the weak bidder's distribution, F_w, horizontally to the right. In the second model, F_s is obtained by "stretching" F_w. In either case, the strong bidder is more likely to have a high willingness to pay. In both models, the FPA dominates the SPA under certain curvature assumptions. However, the SPA dominates in their third model,

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[1] Vickrey (1961), Lebrun (1996), and Cheng (2006) analytically compare revenue in models where bidders draw types from restricted classes of power distributions. Cheng (2010) contains additional examples. Gavious and Minchuk (forthcoming) study auctions with "small" asymmetries. Maskin and Riley (1985) and Doni and Menicucci (2013) assume distributions are discrete. Maskin and Riley's (2000) paper is discussed momentarily. The numerical literature dates back to Marshall et al. (1994) and also includes Fibich and Gavious (2003), Li and Riley (2007), and Gayle and Richard (2008).

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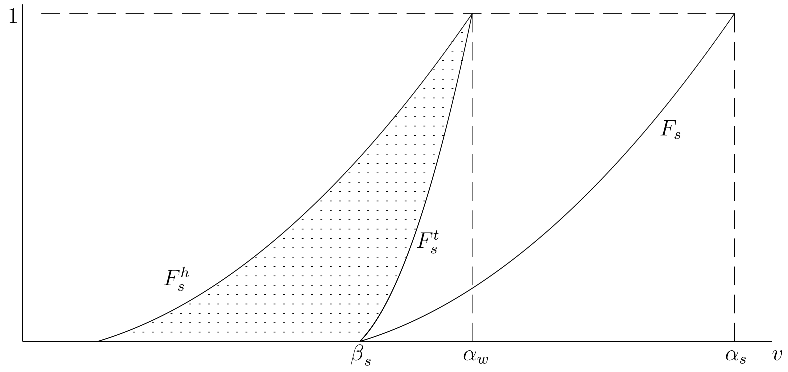

Fig. 1. Truncations and horizontal shifts. Note: F_s has support $[\beta_s, \alpha_s]$. Consider some F_w whose support ends at $\alpha_w \in (\beta_s, \alpha_s)$. Generate the truncation, F_s^t, and horizontal shift, F_s^h, of F_s, such that these end at α_w. The FPA dominates the SPA if F_w is more disperse than F_s^t but less disperse than F_s^h.

in which F_s and F_w share the same support. A central result in the current paper takes Maskin and Riley's (2000) first two models as "benchmarks" and then proves that the FPA is superior whenever the auction environment "lies between" the two benchmarks in a natural way. Several results of this type are presented.

More concretely, consider for now the most stringent assumptions in Maskin and Riley (2000), namely that the distributions are convex and log-concave. An alternative way, used from now on, of thinking about their "stretch" model is that F_w is a truncation of F_s, which I denote F_s^t.[2] In their "shift" model, F_w is a horizontal, left-ward shift of F_s, denoted F_s^h. These cases are depicted in Fig. 1. Of course, holding F_s fixed, F_w can take many other forms. Here, the FPA is shown to dominate if F_w is between F_s^t and F_s^h and satisfies certain regularity conditions. These conditions are satisfied if F_w is more disperse than F_s^t, but less disperse than F_s^h. Similar results obtain if F_s is concave, but with a vertical shift of F_s taking the place of the horizontal shift. A related result for non-monotonic densities is also derived. In this case, another stochastic order of spread, the star order, is more useful than the dispersive order.

The analysis takes as its starting point a new revenue ranking result, due to Kirkegaard (2012a). Kirkegaard (2012a) proves a theorem establishing the superiority of the FPA under two conditions. First, F_s must dominate F_w in terms of the reverse hazard rate. This assumption allows some inferences on bidding behavior in the FPA. The second assumption is, roughly speaking, that F_s is flatter and more disperse than F_w. Maskin and Riley's (2000) first two models satisfy these conditions. While the idea is to invoke Kirkegaard's (2012a) theorem, the challenge remains to describe, in a simple manner, environments where both, possibly contradictory, conditions are satisfied simultaneously. Such an endeavor necessitates a better understanding of the link between the dispersive order and reverse hazard rate dominance.

It is easy to show that reverse hazard rate dominance does not apply if F_w lies below F_s^t in Fig. 1. However, under mild conditions, I show that if F_w is more disperse than F_s^t – which implies that F_s^t is below, and thus first order stochastically dominates, F_w – then F_s dominates F_w in terms of the reverse hazard rate (Lemma 1). That is, there is an intimate relationship between the dispersive order, truncations, and reverse hazard rate dominance. Although results of a similar flavor (but with different assumptions) have appeared in the statistics literature before, this appears to be the first time result of this nature have been used in economics. Section 2 describes why such results may be of more general use in mechanism design problems, beyond the present revenue ranking exercise.

While F_s^t is a useful benchmark, it turns out that the second condition in Kirkegaard's (2012a) theorem is violated if F_w is too far above F_s^t. In fact, when F_s is convex, the condition is violated if F_w is ever above F_s^h in Fig. 1. Thus, Maskin and Riley's (2000) examples are essentially on opposite boundaries of Kirkegaard's (2012a) theorem. In this paper, I develop regularity assumptions that are sufficient to conclude that the FPA is revenue superior to the SPA if F_w falls between F_s^t and F_s^h.

The dispersive order has recently attracted some attention in the theoretical auction literature.[3] Jia et al. (2010), Katzman et al. (2010), and Szech (2011) examine comparative statics in symmetric auctions when bidders' distributions become more disperse. Ganuza and Penalva (2010) consider auctions in which the seller influences the precision of bidders' information by making their signals more or less disperse. In asymmetric auctions, the dispersive order plays a role in determining the qualitative features of revenue-enhancing interventions in particular auction formats, as demonstrated by Kirkegaard (2012b) and Mares and Swinkels (2011a, 2011b). Hopkins (2007) examines bidding behavior in auctions where distribution functions cross and one is smaller than the other in the dispersive order. However, apart from Kirkegaard (2012a), the current paper is the first to explicitly use the dispersive order to rank revenue across standard auctions with asymmetric bidders.

[2] Defining F_w as a truncation of F_s is arguably a more parsimonious way of describing the same setting. In either case, F_s is a multiple of F_w on the shared support, but when F_s is thought of as a stretch of F_w one has to also formulate an extension of F_s on the rest of its support. This leads Maskin and Riley (2000) to impose unnecessarily strong assumptions; see Kirkegaard (2012a).

[3] The dispersive order is also of relevance in the theory of choice under uncertainty. See e.g. Chateauneuf et al. (2004).