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Reclaim-proof allocation of indivisible objects

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A B S T R A C T

We study desirability axioms imposed on allocations in indivisible object allocation problems. The existing axioms in the literature are various conditions of robustness to blocking coalitions with respect to agents' *ex ante* (individual rationality and group rationality) and *ex post* (Pareto efficiency) endowments. We introduce a stringent axiom that encompasses and strengthens the existing ones. An allocation is *reclaim-proof* if it is robust to blocking coalitions with respect to any conceivable *interim* endowments of agents. This is an appealing property in dynamic settings, where the assignments prescribed by an allocation to be implemented need to be made in multiple rounds rather than all in one shot. We show that an allocation is reclaim-proof if and only if it is induced by a YRMH–IGYT mechanism (introduced by Abdulkadiroğlu and Sönmez, 1999) and if and only if it is a competitive allocation.

1. Introduction

We study the classical problem of allocating a set of n indivisible objects, H, to a set of n agents, A. Each agent needs precisely one object, her preferences over objects are *strict*, and monetary transfers are not allowed. An *allocation* is a bijection from A to H (i.e., it specifies for each agent her assigned object). There are many real-life applications of this problem, such as the allocation of offices to faculty members, spots at public schools to students, dormitory rooms to college students, and organs for transplant to patients. We follow the convention in the literature and refer to our objects as "houses."

There are three variants of this problem in the literature, varying in terms of agents' ex ante (initial) endowments. In a *housing market*, each house is initially owned by a distinct agent (Shapley and Scarf, 1974); in a *house allocation problem*, every house is initially unowned (Hylland and Zeckhauser, 1979); and in the general case, a *house allocation problem with existing tenants*, a house is initially either unowned or owned by a distinct agent (Abdulkadiroğlu and Sönmez, 1999).

A *mechanism* (or an *allocation rule*) is a systematic rule to select an allocation for any preference profile that may be reported by agents. Two questions are important in judging the desirability of a mechanism: Does it select a "desirable allocation" for any reported preference profile? And for the previous question to bear significance, does it induce agents to report their preferences honestly? The content of this paper relates to the first question: We disregard issues of agents' strategic behavior, focus on axioms defining a desirable allocation and study their links to the popular mechanisms in this literature.

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An *endowment function* ω maps H to $A \cup \{a_0\}$, where a_0 stands for the social planner, and such that only the social planner can be endowed with multiple houses (i.e., for $a \in A$, $|\omega^{-1}(a)| \leqslant 1$). Throughout, a house owned by the social planner should be understood, and is sometimes referred to, as an unowned house. Two endowment functions stand out for consideration when an allocation μ is to be implemented: the ex ante endowment function, denoted by ω_{ante}, which derives from the ex ante (initial) distribution of houses (i.e., before any assignment prescribed by μ has been made), and the ex post endowment function, denoted by $\omega_{\mu,post}$, which derives from the ex post (eventual) distribution of houses (i.e., after μ has been fully implemented; hence $\omega_{\mu,post} = \mu^{-1}$).

The desirability axioms considered in the earlier literature are various conditions of robustness to "blocking coalitions." A coalition of agents *blocks* an allocation μ with respect to an endowment function ω if there is a way in which coalition members can trade their endowments under ω such that their induced coalitional allocation Pareto dominates their coalitional allocation under μ. The axioms considered in the earlier literature vary with respect to the permissible size of a blocking coalition (one or arbitrary) and the reference endowment function (ω_{ante} or $\omega_{\mu,post}$).

- In the housing market context, the prominent axiom is "group rationality," which is an allocation's robustness to blocking coalitions of any size with respect to the ex ante endowment function. Formally, an allocation μ is *group-rational* if no coalition of agents blocks μ with respect to ω_{ante}.[1] Shapley and Scarf (1974) showed that in a housing market Gale's *top trading cycle* (TTC) mechanism (described in Section 2.2) induces a group-rational allocation; Roth and Postlewaite (1977) later showed that the allocation induced by TTC is indeed the unique group-rational allocation and also the unique competitive allocation.[2]
- In the house allocation problem context, the prominent axiom is "Pareto efficiency," which is an allocation's robustness to blocking coalitions of any size with respect to the associated ex post endowment function. Formally, an allocation μ is *Pareto-efficient* if no coalition of agents blocks μ with respect to $\omega_{\mu,post}$. Svensson (1994) showed that in a house allocation problem the set of Pareto-efficient allocations coincides with the set of allocations induced by the class of *serial dictatorship* mechanisms (also known as *priority rules* or *queue allocation rules*, described in Subsection 2.2).[3]
- In the house allocation problem with existing tenants context, as properties of a desirable allocation, Abdulkadiroğlu and Sönmez (1999) considered two axioms—Pareto efficiency and "individual rationality." Individual rationality is an allocation's robustness to *blocking individuals*, i.e., blocking coalitions of size one, with respect to the ex ante endowment function; hence, it is a weaker requirement than group rationality. Formally, an allocation μ is *individually rational* if no agent strictly prefers the house that she initially owns to the house that she is assigned under μ. They introduced the class of *You request my house–I get your turn* (YRMH–IGYT) mechanisms (described in Subsection 2.2), which induce Pareto-efficient and individually rational allocations. The class of YRMH–IGYT mechanisms is a generalization of TTC and serial dictatorships: This class reduces to TTC in a housing market and to the class of serial dictatorships in a house allocation problem.[4]

We introduce a stringent desirability axiom that subsumes and strengthens group rationality and Pareto efficiency. The following simple example will be useful for our discussion.

Example 1. The houses h_1, h_2, and h_3 are to be allocated to agents a_1, a_2, and a_3. Agents' strict preference rankings of houses are as given in the table below. Initially h_1 and h_2 are unowned (i.e., owned by the social planner, a_0) and h_3 is owned by a_3 (shown in boldface in the table in a_3's column). The allocation π is as displayed in boxes in the table, i.e., π assigns a_1, a_2, and a_3 to h_3, h_1, and h_2, respectively. □

a_1	a_2	a_3
$\boxed{h_3}$	h_3	h_1
h_2	$\boxed{h_1}$	$\boxed{h_2}$
h_1	h_2	$\boldsymbol{h}_3$

In Example 1, the allocation π is group-rational and Pareto-efficient, i.e., it is robust to blocking coalitions with respect to both ex ante and ex post endowment functions.[5] Suppose, however, that, while π is being implemented, the assignments that it prescribes are not made all in one shot; instead, first, let a_2 be assigned h_1 as π prescribes, while a_3 continues

[1] What we define here as a group-rational allocation is commonly referred to in the literature on housing market as a *core allocation*. We use a different term in order to avoid confusion, as we consider robustness to blocking coalitions with respect to ex ante endowment function in a wider context, the class of house allocation problems with existing tenants.

[2] See Abdulkadiroğlu and Sönmez (1998), Bird (1984), Ma (1994), Roth (1982), Svensson (1999) for other studies related to TTC.

[3] See Ergin (2000), Satterthwaite and Sonnenschein (1981), Svensson (1999) for other studies related to serial dictatorships; also, see Bogomolnaia and Moulin (2001), Hylland and Zeckhauser (1979) for studies of the house allocation problem in a random setting.

[4] See Chen and Sönmez (2002), Ekici (2011), Sönmez and Ünver (2005, 2010) for other studies related to YRMH–IGYT.

[5] Note that in Example 1 group rationality reduces to individual rationality because the only agent who initially owns a house is a_3.