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Aggregate comparative statics ☆

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ABSTRACT

In aggregative games, each player's payoff depends on her own actions and an aggregate of the actions of all the players. Many common games in industrial organization, political economy, public economics, and macroeconomics can be cast as aggregative games. This paper provides a general and tractable framework for comparative static results in aggregative games. We focus on two classes of games: (1) aggregative games with strategic substitutes and (2) nice aggregative games, where payoff functions are continuous and concave in own strategies. We provide simple sufficient conditions under which positive shocks to individual players increase their own actions and have monotone effects on the aggregate. The results are illustrated with applications to public good provision, contests, Cournot competition and technology choices in oligopoly.

1. Introduction

In aggregative games, each player's payoff depends on her own actions and some aggregate of all players' actions. Numerous games studied in the literature can be cast as aggregative games, including models of competition (Cournot and Bertrand with or without product differentiation), patent races, models of contests and fighting, public good provision games, and models with aggregate demand externalities.[1] In this paper, we provide a simple general framework for comparative static analysis in aggregative games (thus generalizing Corchón, 1994 which is discussed in greater detail below). Our approach is applicable to a diverse set of applications that can be cast as aggregative games and enables us to provide sufficient conditions for a rich set of comparative static results.

We present results for two sets of complementary environments. First, we focus on aggregative games with strategic substitutes. In games with strategic substitutes, each player's payoff function is supermodular in her own strategy and exhibits decreasing differences in her own strategy and the strategy vector of other players. Second, we turn to "nice" aggregative

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[1] For a long list of examples of aggregative games see Alos-Ferrer and Ania (2005). For more specific applications, see *e.g.* Cornes and Hartley (2005a, 2007), Kotchen (2007), and Fraser (2012). Issues of evolutionary stability (Alos-Ferrer and Ania, 2005; Possajennikov, 2003), evolution of preferences (Kockesen et al., 2000), existence and stability (Dubey et al., 2006; Jensen, 2010), and uniqueness of equilibrium (Cornes and Hartley, 2005b) have also been studied fruitfully in the context of aggregative games.

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games, where payoff functions are continuous, concave (or pseudo-concave) in own strategies, and twice continuously differentiable. For such games, we prove a number of results under a condition which we refer to as *local solvability*, which ensures the local invertibility of the backward reply correspondence described further below.

An informal summary of our results from both aggregative games with strategic substitutes and from nice aggregative games is that, under a variety of reasonable economic conditions, comparative statics are "regular" (for example, in Cournot oligopoly, a reduction in the marginal cost increases a firm's output). More precisely, in nice aggregative games with local solvability, a "positive shock" to any subset of players — defined as a change in parameters that increase the marginal payoff of the subset of players — increases the aggregate, and entry of a new player also increases the aggregate. In aggregative games with strategic substitutes, a positive shock to a player increases that player's strategy and reduces the aggregate of the remaining players' strategies, and entry of a new player reduces the aggregate of the strategies of remaining players. In addition, in aggregative games with strategic substitutes, the aggregate varies monotonically with what we call "shocks that hit the aggregator" which are changes in parameters that have a direct (positive) impact on the aggregator. In a separate section (Section 5), we illustrate all of these results in a variety of economic models, highlighting the broad applicability of the methods we propose and adding several new results and insights.

We should emphasize at this point that there is no guarantee in general that intuitive and unambiguous comparative static results should hold in aggregative games. Take an increase in a player's marginal payoff such as a reduction in an oligopolist's marginal cost: Even though the *first-order effect* of such a shock will of course be positive, it is possible that *higher-order effects* go in the opposite direction so that in equilibrium, the player ends up lowering her strategy and the aggregate falls (see Acemoglu and Jensen, 2011 for an example of this kind). In this light, a major contribution of our paper is to provide minimal conditions to ensure that such higher-order effects do not dominate so that comparative statics become "regular". In particular, our first set of theorems shows that such "perverse" outcomes cannot arise in aggregative games with strategic substitutes, and our second set of results establishes that they can be ruled out in nice aggregative games by the local solvability condition mentioned above.

Our paper is related to a number of different strands in the literature. Comparative static results in most games are obtained using the implicit function theorem. The main exception is for supermodular games (games with strategic complements). Topkis (1978, 1979), Milgrom and Roberts (1990) and Vives (1990) provide a framework for deriving comparative static results in such games. These methods do not extend beyond supermodular games.

More closely related to our work, and in many ways its precursor, is Corchón (1994). Corchón (1994) provides comparative static results for aggregative games with strategic substitutes, but only under fairly restrictive conditions, which, among other things, ensure uniqueness of equilibria. In contrast, our comparative static results for aggregative games with strategic substitutes are valid without any additional assumptions. Another similarity between our paper and Corchón (1994) is that both make use of the so-called *backward reply correspondence* of Selten (1970). In an aggregative game, the backward reply correspondence gives the (best-response) strategies of players that are compatible with a given value of the aggregate.[2] In a seminal paper, Novshek (1985) used this correspondence to give the first general proof of existence of pure-strategy equilibria in the Cournot model without assuming quasi-concavity of payoff functions (see also Kukushkin, 1994). Novshek's result has since been strengthened and generalized to a larger class of aggregative games (*e.g.*, Dubey et al., 2006; Jensen, 2010), and our results on games with strategic substitutes utilize Novshek's (1985) construction in the proofs.[3] Our results on nice aggregative games blend the backward reply approach with the equilibrium comparison results reported in Milgrom and Roberts (1994) and Villas-Boas (1997).

An alternative to working directly with backward reply correspondences as we do, is to use "share correspondences" introduced by Cornes and Hartley (2005a). The share correspondence is the backward reply correspondence divided by the aggregate. This transformation of the problem is useful for questions related to uniqueness and existence and can be used for explicitly characterizing the equilibrium and deriving comparative statics directly in certain cases. However, transforming backward reply correspondences in this way does not simplify any arguments in this paper or strengthen any results.[4]

The rest of the paper is organized as follows. Section 2 defines aggregative games, equilibrium, and backward reply correspondences. Section 3 provides the general comparative static results for aggregative games with strategic substitutes.

[2] The first systematic study of aggregative games (German: *aggregierbaren Spiele*) can be found in Selten (1970). After defining aggregative games, Selten proceeds to define what he calls the *Einpassungsfunktion* (Selten, 1970, p. 154), that is, the backward reply function of an individual player. As Selten proves, the backward reply correspondence is single-valued (a function) provided that the player's best-response function has slope greater than −1. The assumptions imposed by Corchón (1994) imply that the slope of players' best-response functions lie strictly between −1 and 0, so that the backward reply correspondence is both single-valued and decreasing. Neither is necessarily the case in many common games and neither is imposed in this paper.

[3] Novshek's explicit characterization of equilibria is similar to the characterization of equilibrium in supermodular games that uses the fixed point theorem of Tarski (1955). Both of these enable the explicit study of the behavior of "largest" and "smallest" fixed points in response to parameter changes. Tarski's result is used, for example, in the proof of Theorem 6 in Milgrom and Roberts (1990).

[4] It is straightforward to recast Novshek's original existence argument in terms of share correspondences (by simply dividing through everywhere by the aggregate Q). Similarly one would be able to recast our proofs for games with strategic substitutes in terms of share correspondences, but this does not lead to any simplification. As for our results on "nice" games, these are based on the idea that under the local solvability condition, the aggregate backward reply correspondence will be a continuous single-valued function. This obviously holds for the aggregate backward reply correspondence if and only if it holds for the associated share correspondence (since the latter's values equal the former's divided with Q). But this construction does not simplify or enrich our analysis; it simply restates our results in a somewhat different language.