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Multi-item Vickrey–English–Dutch auctions ☆

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ABSTRACT

Assuming that bidders wish to acquire at most one item, this paper defines a polynomial time multi-item auction that locates the VCG prices in a finite number of iterations for any given starting prices. This auction is called the Vickrey–English–Dutch auction and it contains the Vickrey–English auction [Sankaran, J.K., 1994. On a dynamic auction mechanism for a bilateral assignment problem. Math. Soc. Sci. 28, 143–150] and the Vickrey–Dutch auction [Mishra, D., Parkes, D., 2009. Multi-item Vickrey–Dutch auctions. Games Econ. Behav. 66, 326–347] as special cases. By means of numerical experiments, it is showed that when the auctioneer knows the bidders' value distributions, the Vickrey–English–Dutch auction is weakly faster than the Vickrey–English auction and the Vickrey–Dutch auction in 89 percent and 99 percent, respectively, of the investigated problems. A greedy version of the Vickrey–English–Dutch auction is demonstrated to perform even better in the simulation studies.

1. Introduction

A fundamental insight due to Vickrey (1961) is that any sealed-bid auction mechanism that implements the unique minimum Walrasian equilibrium prices, often referred to as the Vickrey–Clarke–Groves prices (VCG prices for short), satisfies several desirable properties whenever the items are homogeneous and each bidder wish to acquire at most one item. For example, no bidder can gain by strategic misrepresentation, and the auction generates an efficient and individual rational outcome. Demange and Gale (1985) and Leonard (1983) demonstrated that the properties of this sealed-bid auction mechanism hold also when the items are heterogeneous (but still assuming unit-demand bidders).[1] Even if this sealed-bid auction satisfies many desirable properties, it is well-known that bidders often prefer iterative auction mechanisms (Engelberecht-Wiggans and Kahn, 1991; Cramton, 1998). There are many reasons for this. For example, an iterative format does not imply full preference revelation and it creates more transparency in the auctioneer's methods. This insight has motivated a substantial amount of research.

In an early paper, Demange et al. (1986) described an English multi-item auction for heterogeneous items based on the Hungarian method of Kuhn (1955). In this iterative auction format, prices are updated based on information regarding groups of items that are overdemanded. Here, a set of items is overdemanded, at a given price vector, if the number

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[1] A more demanding problem, not considered in this paper, is when bidders' are allowed to demand multiple items. See e.g. Ausubel (2004, 2006), Ausubel and Milgrom (2002), Bikhchandani et al. (2011), Bikhchandani and Ostroy (2002), Conen and Sandholm (2002), Cramton et al. (2006), de Vries et al. (2007), Gul and Staccetti (2000), Mishra and Parkes (2007), and Perry and Reny (2005).

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of bidders demanding only items in the set is greater than the number of items in the set. This is a natural approach since a necessary requirement for reaching the VCG prices is that all overdemanded sets of items are eliminated (Hall, 1935).[2] Even if the iterative auction mechanism in Demange et al. (1986) converges to the VCG prices in a finite number of iterations,[3] the price path from the sellers reservation prices to the VCG prices is not unique as it depends on the specific selection of the overdemanded set of items whose prices are updated for the next iteration. In fact, there are typically a very large number of different paths and it is ex ante not possible to identify the fastest (Andersson and Andersson, 2012).[4] Another fundamental problem with the approach in Demange et al. (1986) and related papers (Andersson et al., 2010; Sun and Yang, 2009, among others) is that the termination criteria require an exhaustive search of all subsets of items. This problem is clearly exponential which is a huge problem if the auction contains many items.

The above two problems with the multi-item auction in Demange et al. (1986) can be overcome by using a modification of the mechanism based on the Ford and Fulkerson (1956) algorithm as demonstrated by Andersson et al. (2010), Sankaran (1994), and Mo et al. (1988). This polynomial time unique path English multi-item auction is called the Vickrey–English auction (VE, henceforth) and it always converges to the VCG prices. VE starts at the sellers reservation prices. At these prices there are no weakly underdemanded sets of items (see footnote 2) by construction. The price increases in VE, prescribed by the Ford–Fulkerson algorithm, guarantee that the family of weakly underdemanded sets stays empty in the ascending process. Because all overdemanded sets of items always are eliminated after a finite number of price increases, as no item is infinitely valuable for any bidder, convergence to the VCG prices follows (again, see footnote 2).

The descending counterpart of the English format is the Dutch auction. Such a polynomial time unique path multi-item auction is defined in Mishra and Parkes (2009). This mechanism, called the Vickrey–Dutch auction (VD, henceforth), also identifies the VCG prices in a finite number of iterations. Convergence to the VCG prices follows by symmetrical arguments as in the above, i.e., VD starts at the upper bound of the price space where there are no overdemanded sets of items by construction. The prescribed price decreases then guarantee that the family of overdemanded sets of items stays empty in the descending process while the family of weakly underdemanded sets of items weakly shrinks in each iteration until it is empty.

The main innovation of this paper is the construction of a new polynomial time auction format called the Vickrey–English–Dutch auction (VED, henceforth). The fundamental difference between this mechanism compared to VE and VD is that it is allowed start at an arbitrary vector in the price space and yet locate the VCG prices, i.e., it need not start at the sellers reservation prices, as VE, or at the upper bound of the price space, as VD. Note, however, that VED can start at these prices as any price vector in the price space is an allowed starting point. In this case, VED is identical to VE and VD, and the two latter formats can therefore be regarded as special cases of VED. Note also that VED is not necessarily an ascending or descending format as both price increases and price decreases are allowed in the iterative process.

It is clear that any iterative auction format that converges to the VCG prices generates the same revenue independently of its starting price. Thus, all other things being equal, it is not unreasonable that the auctioneer selects the auction format which has the lowest expected number of iterations before convergence. One motivation for this is the fact that bidders' typically prefer auctions with a shorter running time over auctions with a long running time (Larson and Sandholm, 2001; Parkes et al., 1999). However, in order to evaluate the expected number of iterations, the auctioneer must have a measure of the number of required iterations before convergence when comparing the performance of different auction formats. This paper demonstrates that the measure of the number of iterations between any two price vectors on the path from the starting prices to the VCG prices, for VED, can be based on the Chebyshev metric.

Because VED gives the auctioneer the freedom to start at an arbitrary price vector in the price space, any information regarding the bidders' may help the auctioneer to reduce the expected number of iterations (this information may be the distribution of valuations, bidding behavior in previous auctions, etc.). This is easiest seen by considering the bounding cases of VED, namely VE and VD, and noting that VE (VD) is an ascending (descending) format. It is therefore impossible to decrease (increase) the prices at any iteration. This in built feature of these formats forces the auctioneer to start at the lowest (highest) possible price in the price space to guarantee convergence to the VCG prices. Hence, neither VE nor VD can take advantage of more detailed information about the bidders. For example, if the auctioneer have information about how the valuations of the bidders' are distributed, VE and VD must still start at the lowest and highest possible price in the price space, respectively. For an auctioneer that adopts VED with a flexible starting price, on the other hand, it is easy (e.g. by means of simulations) to find the expected VCG prices and then select this expectation as starting prices. This will obviously reduce the expected number of iterations as illustrated in this paper by means of numerical experiments. Note also that VED always converges to the VCG prices even if the auctioneer does not have any information about the bidders.

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[2] A necessary and sufficient condition for a price vector to be a VCG price vector is that all overdemanded and all weakly underdemanded sets of items are eliminated (Mishra and Talman, 2010, Theorem 2). A set of items is weakly underdemanded, at a given price vector, if the number of bidders that demand some item in the set is weakly lower than the number of items in the set, and the price of each item in the set is strictly higher that the seller's reservation prices (see Definition 3).

[3] This result holds if bidders report truthfully in the iterative process. However, truthful bidding is not a dominant strategy even if the mechanism converges to the VCG prices. This is in sharp contrast to its sealed-bid counterpart. However, truthful revelation constitutes a Nash equilibrium given that certain "activity rules" are imposed. See, e.g., Ausubel (2006), de Vries et al. (2007), Gul and Staccetti (2000), or Parkes (2001) a for detailed discussion and analysis. See also Ausubel (2006).

[4] All possible paths from the sellers reservation prices to the VCG prices, for the family of English auctions with unit-demand bidders, are characterized in Andersson et al. (2010).