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Garbling of signals and outcome equivalence[☆,☆☆]

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ABSTRACT

In a game with incomplete information players receive stochastic signals about the state of nature. The distribution of the signals given the state of nature is determined by the information structure. Different information structures may induce different equilibria. Two information structures are *equivalent* from the perspective of a modeler, if they induce the same equilibrium outcomes. We characterize the situations in which two information structures are equivalent in terms of natural transformations, called *garblings*, from one structure to another. We study the notion of 'being equivalent to' in relation with three equilibrium concepts: Nash equilibrium, agent normal-form correlated equilibrium and the belief invariant Bayesian solution.

1. Introduction

Players are often engaged in a strategic interaction that involves unknown parameters. Following Harsanyi's seminal work, such situations are regarded as games with incomplete information. In particular, information about an unknown parameter is modeled as a stochastic signal whose distribution depends on this parameter. Thus, beyond the action sets and the payoff functions, the description of the game must include an information structure, which determines the distribution of the signals received by the players.

The phrase 'outcome of the interaction' refers to a pair consisting of a state and an action profile which ultimately determine the payoffs. Two information structures are outcome-equivalent with respect to an equilibrium concept, if both yield the same set of outcomes, meaning that they induce the same set of distributions over state-action pairs. Whether the modeler chooses one information structure or another, equivalent to it, the predictions he makes about the outcomes are the same.

We examine when two information structures are equivalent with respect to Nash equilibrium and to two variations of correlated equilibrium that have been studied in a seminal paper of Forges (1993): agent normal-form correlated equilibrium and Bayesian solution. As we shall see, these solution concepts rely on the same incentive compatibility property, and differ only in the joint distributions of signals and actions they allow.

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To illustrate the motivation of our study consider the following scenario. An economist wishes to analyze a situation in which two players jointly have sufficient information to deduce the state of the economy but neither knows anything about it individually. This situation lends itself to various ways of modeling that use distributions over the states and private signals. The economist, the modeler, selects one. This selection, together with the other components of the game (actions, utility functions, etc.) and the equilibrium concept he employs, determine the modeler's prediction about the outcome of the interaction. The question arises as to how sensitive the outcomes are to the modeling selection. Stated differently, what are the implications on the modeler's predictions of his (possibly arbitrary) choices when he explicitly delineates the tiny details of the information structure?

We view Harsanyi's stochastic structure as a modeling tool which does not necessarily correspond to a distribution of 'real world' signals. We focus only on informational issues and assume that all other components of the games (e.g., actions, payoffs, space of states of nature, and even the identity of the players) are unambiguously known, and do not require a modeling selection.

In this paper we shall view Harsanyi's stochastic structure as a modeling tool although it does not necessarily correspond to a distribution of 'real world' signals. We shall focus only on informational issues and assume that all other components of the games (e.g., actions, payoffs, space of states of nature, and even the identity of the players) are unambiguously known, and do not require a modeling selection.

This paper is motivated also by Blackwell's study of comparison of information structures in single-player decision problems under uncertainty (Blackwell, 1953). Blackwell introduces two partial orders over the set of information structures. The first is operational: it compares two structures with respect to attainable state-contingent payoffs in different decision problems. The second order is purely probabilistic. It is formulated in terms of transformations from one structure to another that involve adding noise to the signals. Blackwell proved the equivalence of these two orders. Following Marschak and Miyasawa (1968), a transformation between information structures is called *garbling*. This term alludes to the fact that such a transformation typically entails a loss of information. In the single-player setup Marschak and Miyasawa (1968) have shown that Blackwell's operational order can also be formulated in terms of the optimal payoff a decision maker can guarantee in different problems. They show that one structure yields a higher optimal payoff than another in every decision problem if and only if the latter structure is a garbled version of the former.

The simplest example of a garbling is changing the names of the signals. Recall that as part of choosing the information structure, the modeler chooses the set of signals that the player receives. Clearly, the names of the signals convey no information and changing them has no informational implication. Indeed, changing the names that the modeler chooses for the signals is an operation any player can do on her own. While changing names is a deterministic transformation, general garblings allow also stochastic operations. But they are motivated by the same idea: these are operations that the players can perform on their own over the signals and as a result render the modeler's choice inconsequential.

The multiplayer case is more intricate. The reason is that additional information may eliminate equilibria and consequently be detrimental for the players. This phenomenon has been famously demonstrated by Hirshleifer in the example of revelation of information in insurance markets (Hirshleifer, 1971). While Blackwell's single-player case uses one type of garbling, the intricacy of the multiplayer case gives rise to several natural types of garblings. Each type is described by a particular set of restrictions imposed over the operations performed on signals. Our results associate various solution concepts with different classes of garblings. That is, for any solution concept we specify a type of garbling that makes two information structures outcome-equivalent with respect to this concept.

Outcome equivalence of information structures reflects the modeler's perspective: the two structures are equivalent ways of modeling. On the other hand, garbling reflects the players' perspective: the players can transform the situation represented by one structure to a situation represented by another. Thus, the results of this paper can be interpreted as establishing a similarity between the perspective of the modeler and that of the players.

In an earlier paper (Lehrer et al., 2010) we studied games with common interests, where more information is always advantageous. In that paper, we introduced the definition of garbling in multiplayer setup and proved an analogue of Blackwell's Theorem for games with common interests. We showed that an information structure supports equilibria that induce payoffs lower than those induced by another structure if and only if the former structure is a garbled version of the latter. In the current paper we show that in a multiplayer setup, even without the restriction of common interests, one corollary of Blackwell's argument can still be recovered: two information structures are outcome-equivalent if and only if they give the players the same information about the state of nature.

In a related recent paper Bergemann and Morris (2010) study a solution concept for which more information can eliminate equilibria, but not create new ones. With this notion, they prove that if one structure is a garbled version of another then the latter has a smaller set of equilibrium outcomes than the former. In another close paper, Bergemann and Morris (2013) characterize the set of Bayes correlated equilibria in a class of continuum player games with quadratic payoffs and normally distributed uncertainty.

In addition to games with common interests, there is another family of games in which more information has a positive effect on payoffs: zero-sum games. In this case more information is always advantageous to the player and disadvantageous to the opponent. Pęski (2008) proved an analogous result to Blackwell's Theorem in zero-sum games. Gossner and Mertens (2001) proved that two information structures are equivalent in zero-sum games if and only if they induce the same distribution over hierarchies of beliefs.