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Note

A folk theorem for Bayesian games with commitment ☆

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ABSTRACT

The set of all Bayesian–Nash equilibrium payoffs that the players can achieve by making conditional commitments at the interim stage of a Bayesian game coincides with the set of all feasible, incentive compatible and interim individually rational payoffs of the Bayesian game. Furthermore, the various equilibrium payoffs, which are achieved by means of different commitment devices, are also the equilibrium payoffs of a universal, deterministic commitment game.

1. Introduction

Let us assume that, before making simultaneous strategic decisions, privately informed players can sign binding agreements. Can we characterize the set of all payoffs that they can reach in this way? Myerson (1991, Chapter 6) gives an answer by assuming that agreements are implemented by a mediator: the payoffs are the ones that are achieved by means of a random mechanism satisfying "informational incentive constraints" and "general participation constraints". However, as pointed out by Tennenholtz (2004) and more recently by Kalai et al. (2010), even in the case of two players with complete information, natural commitments may be conditional, so that some care is needed to avoid circularities. Consider for instance the price competition between two sellers; if every seller posts a price and commits to undercut his competitor's price by some amount, the outcome of the commitment strategies might not be well-defined.

Kalai et al. (2010) propose a simple model which overcomes the difficulties. Given a two-person strategic form game G, they allow each player to choose an arbitrary commitment device. In order to account for conditional commitments, they assume that the device of each player can be identified with a response function, which associates an action to every commitment device of the other player. This definition of response functions, taking values in the set of the player's actions, eliminates the possibility of circular or endless reasoning. Every pair of sets of commitment devices and associated response functions $\mathcal{D} = (D_1, (r^1_{d_1})_{d_1 \in D_1}, D_2, (r^2_{d_2})_{d_2 \in D_2})$ transforms the original game G into a commitment game $G(\mathcal{D})$ extending G. It is understood that participation in $G(\mathcal{D})$ is voluntary, in the sense that, in $G(\mathcal{D})$, every player may decide not to commit and just choose (by himself) an action in G.

Kalai et al. (2010)'s main result can be stated as follows: the set of all Nash equilibrium payoffs that can be achieved in some commitment game $G(\mathcal{D})$ extending G coincides with the set of feasible and individually rational payoffs of G. As a by-product, Kalai et al. (2010) construct a universal commitment game extending G, in which all these Nash equilibrium

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payoffs can be achieved at once. Finally, the results go through in the case of n players if individual rationality for a player is formulated in terms of correlated strategies for the others. Tennenholtz (2004) establishes a result that is similar in spirit, but formalizes the players' commitment possibilities as computer programs. His definition of feasible and individually rational payoffs is more restrictive than Kalai et al. (2010)'s one.[1]

In this note, we extend Kalai et al. (2010)'s result to n-person games with incomplete information, namely Bayesian games. A relevant question is then the stage at which the players sign binding agreements: ex ante or interim. We follow the well-founded tradition according to which players make commitments after having learnt their types, namely, at the interim stage (see, e.g., Myerson, 1991). This assumption allows us to describe the commitment *possibilities* of the players in a *Bayesian* game G in the same way as in the case of complete information, i.e., *exactly* as in Kalai et al. (2010). In other words, the *set* of commitment devices (or programs, or instructions to a mediator) that are available to a player is described independently of his private information. A natural justification for this model is that the authority (or the computer) implementing the commitments (or the programs) does not know to which extent the players have private information. Of course, every player chooses his effective commitment device as a function of his type. Being modeled in the same way as in Kalai et al. (2010), our commitments can be conditional but do not give rise to any circularity.

We show that the set of *all* Bayesian–Nash equilibrium payoffs that can be achieved in *some* commitment game $G(\mathcal{D})$ extending a given n-person Bayesian game G coincides with the set $\mathcal{F}(G)$ of feasible, incentive compatible and interim individually rational payoffs of G. As mentioned above, this result is suggested in Myerson (1991, Section 6.6), who gives precise definitions of the previous three properties.[2] However, Myerson (1991) formulates the players' contracting possibilities in terms of *joint random* mechanisms, without explicitly addressing the issue of conditional commitments. As we will explain in detail in Section 2, *deterministic* response functions are a key ingredient to avoid circularities in Kalai et al. (2010)'s approach.

Our result states that two sets coincide and can thus be decomposed into two parts. First, we allow the players to use arbitrary commitment devices $\mathcal{D}$ at the interim stage of G and we establish (in Proposition 1) that every Bayesian–Nash equilibrium payoff of the associated commitment game $G(\mathcal{D})$ is feasible, incentive compatible and interim individually rational, namely, is in $\mathcal{F}(G)$. Second, in Proposition 2, we construct a universal *deterministic* commitment game $G(\mathcal{D}^*)$ extending G such that every payoff in $\mathcal{F}(G)$ (possibly making use of a joint, type-dependent, correlated random device) can be achieved as a Bayesian–Nash equilibrium of $G(\mathcal{D}^*)$.

Peters and Szentes (2012) argue that Tennenholtz (2004) and Kalai et al. (2010)'s approach precludes infinite regress at the outset and is not robust to contractual innovation. Their criticisms apply in an even more severe way to the present paper, which deals with incomplete information. As in Kalai et al. (2010), we identify commitment devices with response functions, which indeed avoid circularities by definition. Contracts are flexible only to the extent that the sets of devices are not restricted at all. More importantly, our commitment games $G(\mathcal{D})$ involve a *single decision stage*, in which every player chooses a commitment device in $\mathcal{D}$.

On the contrary, Peters and Szentes (2012) consider a *two-stage* contracting game.[3] In the first stage, the players offer conditional contracts, which restrict their action sets as a function of the other players' contracts. In the second stage, the players take actions in the restricted sets. Peters and Szentes (2012) impose two requirements on contract spaces: "cross-referentiality" and "invariant punishment". Very loosely speaking, the first requirement captures Tennenholtz (2004)'s idea that contracts can recognize each other and the second one expresses that a player can commit to any of his pure action, as in the voluntary commitment space of Kalai et al. (2010). Peters and Szentes (2012) rely on Gödel's codes to establish that there exists a (well-behaved) contract space which satisfies their requirements. They also show on an example (Section 6.3) that the set of (mildly refined) Nash equilibrium outcomes of their two-stage game can be smaller than the set $\mathcal{F}(G)$ of feasible, incentive compatible and interim individually rational payoffs of the initial Bayesian game G.[4]

While it is not obvious that well-behaved conditional commitments should necessarily lead to two decision stages,[5] it will be rather intuitive that some solutions of our one-shot commitment games cannot be achieved as a (mildly refined) Nash equilibrium of Peters and Szentes (2012)'s two-stage game, since our one-shot games greatly facilitate commitment to punishment against a player who does not commit himself.[6] We believe that such strong forms of conditional commitment may be desirable and achievable in some economic environments, as, e.g., the new markets created by the Internet (this was already a basic motivation in Tennenholtz, 2004; see Ashlagi et al., 2009, for an application involving incomplete information).[7]

[1] Tennenholtz (2004) defines both feasible and individually rational payoffs in terms of mixed strategies, while Kalai et al. (2010) allow the players to use correlated ones. In their Section 5.4, Kalai et al. (2010) make a thorough comparison between their approach and Tennenholtz (2004)'s program equilibria.

[2] In particular, the interim individual rationality condition of a player is formulated in terms of his vector payoff, namely the payoff of his various possible types.

[3] Another difference is that Peters and Szentes (2012) focus on pure strategies. While mixed strategies are essential in our proof of Proposition 2, they do not explain the differences in the results.

[4] Incomplete information is crucial in this example. Under complete information, Peters and Szentes (2012) recover all feasible, individually rational payoffs (in pure strategies).

[5] We refer the reader to Peters and Szentes (2012)'s detailed discussion, which covers various aspects of what can be meant by "well-behaved contractible contracts".

[6] See the end of Section 2.2 for more details.

[7] See Kalai et al. (2010) and Peters and Szentes (2012) for further references on conditional contracts.