Games and Economic Behavior 76 (2012) 329–339

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Existence, incentive compatibility and efficiency of the rational expectations equilibrium ☆

Yeneng Sun [a], Lei Wu [b], Nicholas C. Yannelis [c,d,*]

[a] Department of Economics, National University of Singapore, 1 Arts Link, Singapore 117570, Singapore
[b] Department of Mathematics, National University of Singapore, 2 Science Drive 2, Singapore 117543, Singapore
[c] Department of Economics, Henry B. Tippie College of Business, University of Iowa, Iowa City, IA 52242, United States
[d] Economics, School of Social Sciences, The University of Manchester, Oxford Road, Manchester M13 9PL, UK

ARTICLE INFO

Article history:
Received 23 December 2010
Available online 23 May 2012

JEL classification:
C70
D50
D82

Keywords:
Asymmetric information
Rational expectations equilibrium
Efficiency
Incentive compatibility
Conditional independence
Exact law of large numbers

ABSTRACT

The rational expectations equilibrium (REE), as introduced in Radner (1979) in a general equilibrium setting *à la* Arrow–Debreu–McKenzie, often fails to have desirable properties such as universal existence, incentive compatibility and efficiency. We resolve those problems by providing a new model which makes the REE a desirable solution concept. In particular, we consider an asymmetric information economy with a continuum of agents whose private signals are independent conditioned on the macro states of nature. For such an economy, agents are allowed to augment their private information by the available public signals. We prove the existence, incentive compatibility and efficiency for this new REE concept.

1. Introduction

In seminal papers, Radner (1979) and Allen (1981) extended the finite agent Arrow–Debreu–McKenzie economy to allow for asymmetric information, where each agent is characterized by a random utility function, random initial endowment, and private information with a prior. The equilibrium notion that Radner put forward is called rational expectations equilibrium (REE), which is an extension of the deterministic Walrasian equilibrium of the Arrow–Debreu–McKenzie model. According to the REE, each individual maximizes interim expected utility conditioned on her own private information as well as the information generated by the equilibrium price.

By now it is well known that in a finite agent economy with asymmetric information, a rational expectations equilibrium may not exist[1] (see Mas-Colell et al., 1995, p. 722, for an example due to Kreps), may not be incentive compatible, may not be Pareto optimal and may not be implementable as a perfect Bayesian equilibrium of an extensive form game (see Glycopantis et al., 2005, p. 31, and also Example 9.1.1, p. 43). Thus, if the intent of the REE notion is to capture contracts among agents under asymmetric information, then such contacts not only do they not exist universally in well behaved economies (i.e., economies with concave, continuous, monotone utility functions and strictly positive initial endowments),

☆ We thank the Associate Editor and referees for their helpful comments and suggestions.
* Corresponding author at: Department of Economics, Henry B. Tippie College of Business, University of Iowa, Iowa City, IA 52242, United States.
E-mail addresses: ynsun@nus.edu.sg (Y. Sun), lwu@nus.edu.sg (L. Wu), nicholasyannelis@gmail.com (N.C. Yannelis).
[1] It only exists in a generic sense as Radner (1979) and Allen (1981) have shown.

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2012.05.006

but even if they exist, they fail to have desirable properties, such as incentive compatibility, Pareto optimality and Bayesian rationality.

The main difficulty that one encounters with the REE in a finite agent economy is the fact that individuals are supposed to maximize their interim expected utility conditioned not only on their own private information, but also on the information generated by the equilibrium price; at the same time, the individuals can influence the equilibrium price to their own benefit by manipulating their private information. However, this would not have been a problem if each agent's private information is negligible. This poses the following question. Is it possible to model the REE in such a way that each agent's effect on the equilibrium price is negligible and therefore the REE concept overcomes the difficulties encountered above?

We introduce a new model where the REE concept becomes free of the problems mentioned above. In particular, we consider an asymmetric information economy with a continuum of agents whose private signals are independent conditioned on the macro states of nature. For such an economy, agents are allowed to augment their private information by the available public signals. We call this new notion REE with aggregate signals. However, it is shown in Sun et al. (forthcoming) that it is not true that under a REE in a large economy, the agents will automatically report their signals truthfully. We need to consider those REE prices that capture the meaning of perfect competition, i.e., they depend only on the macro states and are not influenced by individual agents' private information. In this case, incentive compatibility is not an issue since an individual agent's private signal can influence neither the macro states nor the aggregate signals. We then prove the existence and efficiency for this new REE notion. We also show that whenever the equilibrium price fully reveals the macro states, the notion of REE with aggregate signals is equivalent to the notion of REE in the classical sense.

The paper is organized as follows. In Section 2, we present the economic model, the notions of REE, Pareto optimality and incentive compatibility, and some assumptions. The main result is stated in Section 3. In Section 4, we discuss related literature. Some concluding remarks are provided in Section 5. The proofs are given in Appendix A.

2. The economic model

In this section, we define the notion of a private information economy, followed by the definitions of rational expectations equilibrium, rational expectations equilibrium with aggregate signals, Pareto optimality and incentive compatibility.

2.1. Private information economy

We consider an atomless probability space[2] $(I, \mathcal{I}, \lambda)$ as the space of agents. Each agent receives a *private signal* of type $q \in T^0 = \{q_1, q_2, \ldots, q_L\}$.[3] Let $\mathcal{T}^0$ denote the power set of T^0, and $\Delta(T^0)$ the set of all probability distributions on T^0. A *signal profile* t is a function from I to T^0. For $i \in I$, $t(i)$ (also denoted by t_i) is the private signal of agent i while t_{-i} is the restriction of t to the set $I \setminus \{i\}$. Let $(T, \mathcal{T}, P)$ be a probability space that models the uncertainty associated with the private signal profiles for all the agents.[4] For simplicity, we shall assume that $(T, \mathcal{T})$ has a product structure so that T is the product of T_{-i} and T^0, while $\mathcal{T}$ is the product σ-algebra of $\mathcal{T}^0$ and a σ-algebra $\mathcal{T}_{-i}$ on T_{-i}. For $t \in T$ and $t'_i \in T^0$, we shall adopt the usual notation (t_{-i}, t'_i) to denote the signal profile whose value is t'_i for i and t_j for $j \neq i$.

The *private signal process* is a function from $I \times T$ to T^0 such that $f(i,t) = t_i$ for any $(i,t) \in I \times T$. For each $i \in I$, let $\tilde{t}_i$ be the projection mapping from T to T^0 with $\tilde{t}_i(t) = t_i$. The *private signal distribution* τ_i of agent i over T^0 is defined as $\tau_i(\{q\}) = P(\tilde{t}_i = q)$ for $q \in T^0$. $P^{T_{-i}}(\cdot|q)$ is the conditional probability measure on the measurable space $(T_{-i}, \mathcal{T}_{-i})$ when the private signal of agent i is $q \in T^0$. If $\tau_i(\{q\}) > 0$, then it is clear that for $D \in \mathcal{T}_{-i}$, $P^{T_{-i}}(D|q) = P(D \times \{q\})/\tau_i(\{q\})$.

We also would like to include another source of uncertainty in our model – the macro level uncertainty. Let $S = \{s_1, s_2, \ldots, s_K\}$ be the set of all possible macro states of nature, and $\mathcal{S}$ the power set of S. The S-valued random variable $\tilde{s}$ on T models the macro level uncertainty. For each macro state $s \in S$, denote the event $(\tilde{s} = s) = \{t \in T\colon \tilde{s}(t) = s\}$ that s occurs by C_s. The probability that s occurs is $\pi_s = P(C_s)$. Without loss of generality, assume that $\pi_s > 0$ for each $s \in S$. Let P_s be the conditional probability measure on $(T, \mathcal{T})$ when the random variable $\tilde{s}$ takes value s. Thus, for each $B \in \mathcal{T}$, $P_s(B) = P(C_s \cap B)/\pi_s$. It is obvious that $P = \sum_{s \in S} \pi_s P_s$. Note that the conditional probability measure P_s is often denoted as $P(\cdot|s)$ in the literature.

The common *consumption set* for all the agents is the positive orthant $\mathbb{R}^m_+$. Let u be a function from $I \times \mathbb{R}^m_+ \times T$ to $\mathbb{R}_+$ such that for any given $i \in I$, $u(i,z,t)$ is the *utility* of agent i at consumption bundle $z \in \mathbb{R}^m_+$ and signal profile $t \in T$. For any given $(i,t) \in I \times T$, we assume that $u(i,z,t)$ (also denoted by $u_{(i,t)}(z)$ or $u_i(z,t)$)[5] is continuous, monotonic in $z \in \mathbb{R}^m_+$.[6] For each $z \in \mathbb{R}^m_+$, $u_z(\cdot,\cdot)$ is an integrable function on $I \times T$.[7]

[2] We use the convention that all probability spaces are countably additive.

[3] In the literature, one usually assumes that different agents have different sets of private signals and requires that agents receive each of them with positive probability. For notational simplicity, we choose to work with a common set T^0 of private signals, but allow zero probability for some of the redundant signals. There is no loss of generality in this latter approach.

[4] Thus T is a space of functions from I to T^0.

[5] In the sequel, we shall often use subscripts to denote some variable of a function that is viewed as a parameter in a particular context.

[6] The utility function $u(i,\cdot,t)$ is monotonic if for any $y, z \in \mathbb{R}^m_+$ with $y \leqslant z$ and $y \neq z$, $u(i,y,t) < u(i,z,t)$.

[7] The measure structure on the product space $I \times T$ will be specified in Section 2.3.