



Note

Evolutionary stability in repeated extensive games played by finite automata

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ABSTRACT

We discuss the emergence of cooperation in repeated Trust Mini-Games played by finite automata. Contrary to a previous result obtained by Piccione and Rubinstein (1993), we first prove that this repeated game admits two Nash equilibria, a cooperative and a non-cooperative one. Second, we show that the cooperative equilibrium is the only (cyclically) stable set under the so-called best response dynamics.

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1. Introduction

Repeated games have enormous sets of equilibria. In a seminal article, Abreu and Rubinstein (1988) introduced the idea that the equilibrium selection problem could be addressed by modeling strategies as finite automata. In this approach, the total payoff of a strategy is a combination of the complexity of the automaton that represents it (as measured by the number of its states) and the payoff it obtains in the playing of the game. They proved that arbitrarily small costs of complexity could drastically reduce the strategies that can be sustained in equilibrium. In the repeated Prisoners' Dilemma (PD), for example, some popular strategies such as Tit for Tat (TfT) cannot be Nash equilibria. The reason is that, in playing against itself, TfT never reaches the states in which it does not cooperate. It follows that a strategy of unconditional cooperation is a best response to TfT, because it obtains the same payoff as TfT itself, but with a smaller number of states.

Abreu and Rubinstein (1988) proved that cooperation can only be achieved by machines that put the “punishing phase” first. Each machine starts by “punishing” the other by playing Defect for a fixed number of rounds and does not revert to cooperation unless the other machine has played Defect for the same number of rounds. Once the “punishing phase” is over, both machines start cooperating. Switching to defection during the “cooperative phase” is deterred by the threat to start the punishment phase all over again. Abreu and Rubinstein (1988) provide a nice interpretation of this initial phase of punishment as a “show of strength”: at the beginning of the play, each machine will “test” the ability of the other machine to punish an eventual defection. Machines that are unable to “punish” are exploited by unending defection.

While this argument drastically restricts the strategies that one can observe in equilibrium, it still allows for a huge variety of possible outcomes, including perpetual defection. Binmore and Samuelson (1992) used an evolutionary model to study the resulting equilibrium selection problem and obtained a stark result. When the cost of complexity is so small that

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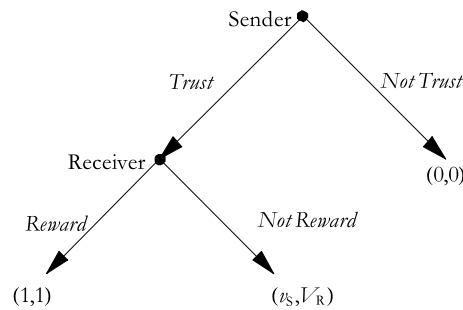


Fig. 1. The Trust Mini-Game.

it can be ranked lexicographically after the game payoffs, the only (modified) evolutionarily stable strategies are those that cooperate throughout the game. However, this result proved to be fragile. Volij (2002) showed that it crucially depends on the way complexity costs are modeled. When they enter directly into a machine's payoff function the opposite conclusion holds true: Defect is the only evolutionarily stable strategy.²

We contribute to this literature by studying the emergence of cooperation in sequential games, and in particular in the Trust Mini-Game (TG) in Fig. 1. It is the sequential version of a PD, in which the first player (the *Sender*) chooses whether to Trust (*T*) the second player (*Receiver*), who in turn decides whether to Reward (*R*) the Sender's trust or not (*NR*). With the assumption that $v_S < 0 < 1 < V_R$, (*NT*, *NR*) is the game's unique Nash equilibrium.

At first sight, cooperation is even less likely to emerge in sequential games than in simultaneous ones. Piccione and Rubinstein (1993) proved that in any sequential game, if players are constrained to choose pure strategies, the only Nash equilibria for the machine game are constant repetitions of one of the equilibria of the stage game. In the TG this entails that there are no pure strategy equilibria other than an infinite repetition of (*NT*, *NR*). The intuition behind their result is as follows. In the TG the punishment phase would consist of a finite number of rounds in which the Sender plays *NT* and the Receiver plays *NR*. However, the Receiver's behavior cannot be observed when the Sender plays *NT*. It follows that the Receiver can eliminate the states associated with the punishment phase (in which he plays *NR*) and reduce the complexity of his strategy. Without the punishment phase, however, cooperation cannot be in equilibrium.

To prove that cooperation can emerge in the repeated TG we have first to show that (contrary to Piccione and Rubinstein, 1993) there can be an NE in which the two players play Trust and Reward. We do this by relaxing the assumption that players are constrained to use pure strategies. This assumption was common in the early literature, where it was justified with the argument that it is unclear how the complexity of a mixed strategy could be modeled (see Abreu and Rubinstein, 1988). However, in the kind of evolutionary models we shall deal with, mixed strategies have an uncontroversial interpretation in terms of polymorphisms within populations of agents, each of whom uses a pure strategy. In these models considering mixed strategies is important because mixed strategy NE may turn out to be stable under some adjustment dynamics.

We first show that once players are allowed to use mixed strategies, Piccione and Rubinstein's (1993) result is overturned: besides a set of non-cooperative equilibria, there is a (mixed strategy) cooperative Nash equilibrium. Second, we address the equilibrium selection problem by using the so-called best response dynamics. The game is assumed to be played repeatedly by agents belonging to two separate populations, one of senders the other of receivers. Occasionally, agents are given the opportunity to change the machine they use to play the repeated game, in which case they adopt the machine that yields the largest payoff. We show that under this adjustment process there is a path that leads from the non-cooperative equilibrium to the cooperative one, but not vice versa. Formally, the cooperative mixed strategy NE is socially stable in the sense introduced by Matsui (1992), while the set of non-cooperative equilibria is not. This shows that Volij's (2002) result is also fragile. In a Prisoner's Dilemma in which one player is allowed to choose first, the non-cooperative equilibrium cannot be stable and cooperation is bound to emerge.

The paper proceeds as follows. Section 2 introduces the necessary technicalities and definitions. Section 3 contains the proof that a cooperative equilibrium exists in the repeated TG. Section 4 introduces the best response dynamics and proves that the cooperative equilibrium is the only stable equilibrium. Section 5 concludes. All proofs are relegated to Appendix A.

2. The model: definitions

Two players repeatedly play the Trust Game in Fig. 1. We shall indicate with $S_S = \{T, NT\}$ and $S_R = \{R, NR\}$ the pure strategy sets for the Sender and the Receiver. The set $E = \{NT, (T, NR), (T, R)\}$ is the set of outcomes of the game. (Note that since the trust game is an extensive form game, E does not correspond to $S_S \times S_R$.) $h_i(e)$ is the payoff that player i obtains on reaching the end-node $e \in E$ ($i = \text{Sender, Receiver}$).

A finite automaton, or a machine, M is a finite collection of states of which one is the initial one. Each state is associated to a strategy, which is the strategy the automaton plays when in that state. After each round the state of the automaton

² Samuelson and Swinkels (2003) discuss the differences between these two approaches.

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