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Note

On the correspondence of contracts to salaries in (many-to-many) matching

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ABSTRACT

In this note, I extend the work of Echenique (2012) to show that a model of many-to-many matching with contracts may be embedded into a model of many-to-many matching with wage bargaining whenever (1) all agents' preferences are substitutable and (2) the matching with contracts model is *unitary*, in the sense that every contractual relationship between a given firm–worker pair is specified in a single contract. Conversely, I show that unitarity is essentially necessary for the embedding result.

Since its introduction, the matching with contracts model has been extensively generalized (Ostrovsky, 2008; Hatfield and Kominers, 2010, 2012), and has been applied in surprising contexts such as cadet–branch matching (Sönmez and Switzer, 2011; Sönmez, 2011) and the Japanese medical residency match (Kamada and Kojima, 2011; Kamada and Kojima, forthcoming).[1] Echenique (2012) has recently shown that under the substitutability condition crucial for many of the results of Hatfield and Milgrom (2005), the matching with contracts model directly embeds into the earlier, and seemingly less general, Kelso and Crawford (1982) model of many-to-one matching with salaries and gross substitutes preferences.

The key insight of Echenique (2012) is that in many-to-one matching with contracts models, contract negotiations between a firm and worker are in a sense "separable" from other contract negotiations whenever that firm and worker have substitutable preferences. In this note, I observe that this insight—and hence the embedding result of Echenique (2012)—applies more generally.

Specifically, I show that bargaining over contracts is isomorphic to a firm–worker salary bargain whenever agents' preferences are substitutable and *a firm and worker are allowed to sign at most one contract with each other*. This latter condition, which I call *unitarity*, may be a natural assumption for (many-to-many) firm–worker matching markets in which each worker can serve in at most one role at each firm.[2] Unitarity is automatic in the many-to-one matching settings considered by Hatfield and Milgrom (2005) and Echenique (2012), as in those settings workers have unit demand for contracts; it is also assumed in the many-to-many matching with contracts model of Klaus and Walzl (2009).[3]

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[1] Hatfield and Milgrom (2005) introduced the matching with contracts model as a generalization of the Kelso and Crawford (1982) model of many-to-one matching with salaries; the possibility of such a generalization was first noted in remarks of Crawford and Knoer (1981) and Kelso and Crawford (1982).

[2] The market designer must exercise care, however, in defining the "roles" that a worker may serve. As Hatfield and Kominers (2010) demonstrated, the most natural contracting model for a matching market may involve splitting jobs into multiple distinct positions (e.g., a night shift and a day shift).

[3] Hence, my result extends the Echenique (2012) approach to the setting of Klaus and Walzl (2009).

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I demonstrate (by example) that unitarity is essentially necessary for the Echenique (2012) embedding result. Nonunitary many-to-many matching models such as that of Hatfield and Kominers (2010) need not correspond to wage bargaining models, even if all agents have substitutable preferences over contracts.

1. Preliminaries

1.1. Basic models

Extending the framework of Echenique (2012), I introduce two models of generalized matching among a (finite) set F of firms and a (finite) set W of workers.

1.1.1. Matching with contracts

A model of (*many-to-many*) *matching with contracts* is specified by a set $X \subseteq F \times W \times T$ of contracts, where T is (finite) a set of contractual terms, along with a (one-to-one) utility function $u_i : 2^X \to \mathbb{R}$ for each $i \in F \cup W$. For a contract $x \in X$, I denote by x_F and x_W the firm and worker associated to x, respectively. For $Y \subseteq X$, I denote by $Y_i \equiv \{x \in Y\colon\ i \in \{x_F, x_W\}\}$ the set of contracts in Y associated to $i \in F \cup W$. Although the utility function of each agent $i \in F \cup W$ is defined (for convenience) over the entire space 2^X, I require that it depend only on the contracts actually available to i, that is, that $u_i(Y) = u_i(Y_i)$ for each $Y \subseteq X$.[4]

The utility function of $i \in F \cup W$ determines a *preference relation* P_i over sets of contracts $Y \subseteq X_i$ and an associated *choice function*

$$C_i(Y) \equiv \max_{P_i}\{Z\colon\ Z \subseteq Y_i\}$$

defined over sets of contracts $Y \subseteq X$.[5] The preferences of $i \in F \cup W$ are *substitutable* if for any $x, z \in X$ and $Y \subseteq X$,

$$x \notin C_i\big(Y \cup \{x\}\big) \implies x \notin C_i\big(Y \cup \{x, z\}\big).^6$$

A set of contracts $Y \subseteq X$ is called an *allocation*. An allocation $Y \subseteq X$ is said to be *individually rational* if $C_i(Y) = Y_i$ for all $i \in F \cup W$, and is said to be *unblocked* if there does not exist a nonempty set of contracts $Z \nsubseteq Y$ such that $Z_i \subseteq C_i(Y \cup Z)$ for all $i \in F \cup W$. An allocation is *stable* if it is both individually rational and unblocked.[7]

1.1.2. Matching with salaries

A model of *(many-to-many) matching with salaries* is specified by a set $S \subseteq \mathbb{R}_+$ of possible salaries, a (one-to-one) utility function $v_f : 2^{\{f\} \times W \times S} \to \mathbb{R}$ for each $f \in F$, and a (one-to-one) utility function $v_w : 2^{F \times \{w\} \times S} \to \mathbb{R}$ for each $w \in W$.[8] Without loss of generality, I restrict S to be finite, and identify it with the set of positive integers $\{1, \ldots, \bar{S}\}$ for some $\bar{S}$ suitably large that no worker is ever hired at salary $\bar{S}$, i.e. such that $v_f\big(B \cup \{(f, w, \bar{S})\}\big) < v_f(B)$ for all $B \subseteq (\{f\} \times (W \setminus \{w\}) \times S)$.

For each $f \in F$, the utility function v_f induces a demand function $D_f : S^{F \times W} \to 2^{\{f\} \times W \times S}$ defined by

$$D_f(s) \equiv \arg\max_{B \subseteq \{f\} \times W \times S}\big(v_f(B)\big).^9$$

The demand function $D_w : S^{F \times W} \to 2^{F \times \{w\} \times S}$ of each worker $w \in W$ is defined analogously. The demand function D_f of $f \in F$ satisfies the *gross substitutes* condition if, for any two salary matrices s and s' with $s_f \leqslant s'_f$,

$$(f, w, s_{fw}) \in D_f(s) \implies \big(f, w, s'_{fw}\big) \in D_f\big(s'\big)$$

for any $w \in W$ for which $s_{fw} = s'_{fw}$. Analogously, the demand function D_w of $w \in W$ satisfies the *gross substitutes* condition if, for any two salary matrices s and s', for which $s_w \geqslant s'_w$,

[4] Note that unlike the many-to-one matching with contracts models considered by Hatfield and Milgrom (2005) and Echenique (2012), my model does not require that the utility functions of workers $w \in W$ exhibit *unit-demand*, i.e. that they be choice-equivalent to utility functions on the restricted space $\{\{x\}\colon\ x \in X_i \cup \{\emptyset\}\}$.

[5] Here, I use the notation $\max_{P_i}$ to indicate that the maximization is taken with respect to the preferences of agent i.

[6] Intuitively, this condition means that there are no two contracts $x, z \in X$ which are sometimes “complements” in the sense that the availability of z makes x more attractive.

[7] A number of alternative stability concepts are available for many-to-many matching settings (Blair, 1988; Echenique and Oviedo, 2006; Klaus and Walzl, 2009). In general, the choice of solution concept is somewhat immaterial for the present exercise: Embedding results analogous to Theorem 2 hold so long as the stability concept under consideration for the case of matching with salaries corresponds to that considered for the case of matching with contracts. Nevertheless, I fix a choice of stability concept for concreteness, using that of Hatfield and Kominers (2010, 2012), which is neither weaker nor stronger than the *setwise stability* concept of Echenique and Oviedo (2006).

[8] I use the convention that utility functions for models of matching with contracts are denoted u, while those for models of matching with salaries are denoted v.

[9] Note that the demand of f only depends upon the salaries s_{fw} of workers w at f.