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Note

Competitive problem solving and the optimal prize schemes ☆

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ABSTRACT

Agents compete to solve a problem. Each agent simultaneously chooses either a safe method or a risky method to solve the problem. This paper analyzes a prize scheme as an incentive to induce the optimal risk-taking level which maximizes the designer's interest. It is shown that whenever the winner-take-all scheme induces excessive risk-taking, there exists a prize scheme which induces the optimal risk-taking. Moreover, the existence of such a prize scheme is guaranteed if the number of competitors is sufficiently large.

1. Introduction

In economics, a contest is usually studied as an incentive scheme to promote a higher level of investment/effort. However, for some practitioners, a contest is an incentive scheme to induce risk-taking. For example, the X-prize foundation, one of leading innovation prize organizers, emphasizes the importance of risk-taking in innovation and claims that innovation prizes are an effective incentive scheme to promote risk-taking. In fact, in R&D races, whether to take a risk is often a more important decision than how much to invest. In the Human Genome Project, National Institute of Health (NIH) and Celera Genomics competed for determining the sequence of chemical based pairs for human DNA. NIH employed a well-known safe method and Celera Genomics employed a risky method whose effectiveness was unknown. In spite of its small budget, Celera Genomics won the race and, consequently, the "risky method" became a popular sequencing method.

On the other hand, the winner-take-all scheme is not always an effective incentive scheme. For instance, suppose that a bonus is awarded only to a worker whose sales reach a certain level earlier than other workers. When all workers try a new sales approach to outperform others, the firm's expected sales can be lower than in the case where all salespersons use established methods. Hence, the prize designer has to employ an appropriate prize scheme depending on the risk-taking level he wants to induce. This paper analyzes prize schemes as an incentive to induce the optimal risk-taking. Given an optimal risk-taking level, we provide sufficient conditions for the existence of a prize scheme which induces the optimal level.

In Section 2, we introduce our model. The model consists of a finite number of agents who compete to solve a common problem. Each agent is endowed with a status quo technology which is private information. Each agent then simultaneously decides whether to use the status quo technology, the "safe method" or try a new technology, the "risky method." Concretely,

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the time to find a solution with the safe method is known, while the time to find a solution with the risky method is determined by a random draw. Each agent receives a non-negative prize when he finds a solution and the size of the prize depends only on the ranking of the time he took to find a solution.

In Section 3, we analyze the equilibrium. First, we show that, given any prize scheme, any equilibrium is monotonic in the sense that, whenever there is an agent who uses the risky method, an agent with lower status quo technology uses the risky method. Second, we analyze the equilibrium under the winner-take-all scheme and show that the equilibrium is unique. The winner-take-all scheme induces risk-taking even if the risky method tends to take much longer than the safe method. Furthermore, we show that if the number of agents is sufficiently large, all agents use the risky method in the equilibrium.

Section 4 analyzes the optimal prize scheme which is defined as a prize scheme that induces the optimal risk-taking in an equilibrium. We then provide a condition which guarantees the existence of an optimal prize scheme. Concretely, it is shown that whenever the winner-take-all scheme induces an excessive risk-taking given an optimal risk-taking level, the existence of an optimal prize scheme is guaranteed. The idea of this result is the following. Note that, by rewarding lower ranked agents, the designer can reduce the competitive pressure of the prize scheme. Thus, whenever the competitive pressure of the winner-take-all scheme induces an excessive risk-taking, the designer can induce the optimal level by reducing the competitive pressure of the prize. Since the winner-take-all scheme induces the highest possible level of risk-taking with a sufficiently large number of agents, the existence of an optimal prize scheme is guaranteed if the number of agents is sufficiently large.

Section 5 concludes this paper with a discussion.

1.1. *Related literature*

Since agents compete to find a solution, our model is categorized as a strategic search model. Unlike other strategic search models such as Fershtman and Rubinstein (1997), Dasgupta and Stiglitz (1980) where agents choose search intensity, the agent in our model chooses the level of risk-taking. In this respect, our model is closer to contest models where agents choose the level of risk-taking, e.g., Hvide and Kristiansen (2003), and Krakel (2008).

Our model also belongs to the literature which designs a rank dependent reward scheme as an optimal incentive scheme, e.g., Lazear and Rosen (1981), Taylor (1995), Moldovanu and Sela (2001, 2006), Hvide (2002), Che and Gale (2003), and Rajan and Reichelstein (2006, 2009). Unlike models in which the incentive scheme is designed to induce an optimal effort level, our prize scheme is designed to induce an optimal risk-taking level. Thus, our paper is closer to Hvide (2002). In his paper, two agents compete to achieve a higher performance. Each agent chooses not only his effort level but also the variance of his performance under symmetric information. Hvide then proposes a modified rank dependent incentive scheme which induces the optimal effort level discouraging an excessive risk-taking. On the other hand, in our paper, each agent is endowed with a status quo technology as private information and decides whether to try a new technology. Our main interest is in designing a prize scheme which induces “right types” to take a risk.

Since risk-taking in our model means trying a new technology, we can also interpret prize schemes as an incentive for experimentation. Unlike multi-armed bandit models[1] in which the decision maker has an incentive to try a risky choice because of its dynamic character, our model focuses on the static problem where agents have no incentive to try a new technology without any prize scheme. The optimal prize scheme is designed to induce the optimal experimentation level in such situations.

2. Model

There are I agents, $i = 1, 2, \dots, I$, who compete to solve a problem. When agent i finds a solution at $t_i \in [0, \infty)$ and he is the n-th agent who finds a solution, he receives prize $z_n \in [0, \infty)$ at t_i. A *prize scheme* is then defined as $z \in Z$ where Z is

$$\left\{ (z_1, z_2, \dots, z_n, \dots, z_I) \in [0,1]^I \colon \quad \begin{array}{l} \text{(i) } z_{n+1} \leqslant z_n \text{ for any } n \\ \text{(ii) } \displaystyle\sum_{n=1}^{I} z_n = 1 \end{array} \right\}.$$

In short, Z is the set of prizes such that (i) the prize is higher if the ranking is higher and (ii) the prize budget is 1.

When the agent finds a solution at t and receives prize $x \in \mathbb{R}$, his payoff is determined by $u(x, t)$ which satisfies the following assumptions:

Assumption 1. Given any $t \in [0, \infty)$, $u(x, t)$ is continuous and strictly increasing in x.

Assumption 2. Given any $x \in [0, \infty)$, $u(x, t)$ is continuous and strictly decreasing in t.

[1] For a survey of multi-armed bandit models, see Bergemann and Valimaki (2008).