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Coalitional bargaining games with random proposers: Theory and application ☆

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ABSTRACT

We consider a non-cooperative coalitional bargaining game with random proposers in a general situation for which players differ in recognition probability and time preference. We characterize an efficient equilibrium as the generalized Nash bargaining solution that belongs to the core. The model is applied to wage bargaining between an employer and multiple workers. Although involuntary unemployment may occur in equilibrium, full employment emerges as players become sufficiently patient.

1. Introduction

We consider a non-cooperative bargaining theory for coalition formation. Ray (2007) provides an excellent review of recent work in this growing field. The bargaining game we study in this paper, called the random-proposer model, generalizes the Baron and Ferejohn (1989) model for a majority-rule cake division problem to a coalitional bargaining problem described as an n-person cooperative game.

The bargaining game has the following simple rule of sequential negotiations. At the start of every round, one player is randomly selected (or recognized) as a proposer among all players remaining in the game. A proposal consists of a coalition and a payoff allocation. Once the proposal is agreed by all members, the coalition forms and its members quit the game. Negotiations continue among the remaining players. The game stops when no players remain. Players discount future payoffs.

Baron and Ferejohn (1989) considered the random-proposer model in a simple-majority voting game and established the existence and uniqueness of a stationary subgame perfect equilibrium (SSPE) when voters were identical in recognition probability and discount factors for future payoffs. Since the seminal work, the Baron and Ferejohn model has been extensively studied in the literature on legislative bargaining and has been generalized for various applications. Baron and Kalai (1993) characterized the Baron–Ferejohn equilibrium as the simplest subgame perfect equilibrium of the correspond-

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ing automaton game. Banks and Duggan (2000) proved the existence of an SSPE for a multidimensional model under a general voting rule in a general situation in which players differ in recognition probability and time preference. Eraslan (2002) proved the uniqueness of an SSPE payoff under a q-majority rule in a general case of recognition probability and time preference. Winter (1996) examined the advantage of veto players. Montero (2006) explored the relationship between the nucleolus as a power index and recognition probability. For other studies, see Eraslan and Melro (2002), Laruelle and Valenciano (2008), Montero (2002) and Norman (2002).

Compared to the literature on voting games, there are fewer reports on the random-proposer model for general cooperative games. In an n-person super-additive coalitional game with transferable utility, Okada (1996) demonstrated that there was no delay in agreement for all SSPEs, and characterized an efficient SSPE under uniform recognition probability as the equity allocation in the core of the game when players are patient. Generalizing this result, Yan (2002) proved that every core allocation is sustained as a unique SSPE payoff if it is used as the recognition probability (after normalization). Okada (2000) considered the possibility of renegotiations and showed that efficient payoff allocation is attained in every SSPE after a finite number of renegotiation steps. Gomes (2005) and Gomes and Jehiel (2005) extended the analysis to the case of externality. In all these studies it was assumed that players have common discount factors for future payoffs.

In this note, we extend previous studies on coalitional bargaining with random proposers to a general situation in which players differ in recognition probability and time preference. In the first part, we establish the existence of an SSPE and characterize the efficient SSPE as follows. The expected payoff for player i in the efficient SSPE is proportional to the ratio $p_i/(1-\delta_i)$ where p_i is the recognition probability for player i and δ_i is his discount factor for future payoffs. The efficient SSPE exists if and only if the expected payoffs for players belong to an enlarged set of the core (an ε core). This enlarged core shrinks to the usual core as players become patient. In the special case for which all players have common discount factors, the efficient SSPE payoff converges to the generalized Nash bargaining solution as players become patient.

In the second part, the theory is applied to wage bargaining in a labor market. There are few non-cooperative models of coalitional bargaining in a labor market, apart from those of Stole and Zwiebel (1996) and Westermark (2003). A critical difference between their results and ours is that wage bargaining was formulated as a bilateral bargaining sequence and only full (efficient) employment appears in equilibrium. Stole and Zwiebel (1996) characterized the equilibrium wage as the Shapley value. In contrast to previous studies, we show that involuntary unemployment can occur in equilibrium.[1] We also show that the probability of full employment converges to 1 as players become patient. In the limit, a unique SSPE payoff is characterized as the "constrained" Nash bargaining solution that maximizes a generalized Nash product within the core of the economy.

Our result is closely related to a recent work of Compte and Compte and Jehiel (2010). They prove the existence of an asymptotically efficient, mixed strategy SSPE in a special case that only one coalition forms and all players have identical recognition probability and discount factors for future payoffs, and characterize its limit payoff as the core allocation that maximizes the Nash product. The unique SSPE payoff in our wage bargaining model attains asymptotic efficiency.

2. Preliminaries

We consider an n-person game (N, v) in coalitional form with transferable utility. $N = \{1, 2, \ldots, n\}$ is the set of players. A non-empty subset S of N is called a *coalition* of players. Let $\mathcal{C}(N)$ be the set of all coalitions of N. The *characteristic function* v is a real-valued function on $\mathcal{C}(N)$ satisfying (1) (zero-normalized) $v(\{i\}) = 0$ for all $i \in N$, (2) (super-additive) $v(S \cup T) \geqslant v(S) + v(T)$ for any two disjoint coalitions S and T, and (3) (essential) $v(N) > 0$.

A payoff allocation for coalition S is a vector $x^S = (x_i^S)_{i\in S}$ of real numbers, where x_i^S represents a payoff for player $i \in S$. A payoff allocation x^S for S is *feasible* if $\sum_{i\in S} x_i^S \leqslant v(S)$. Let X^S denote the set of all feasible payoff allocations for S and let X_+^S denote the set of all elements in X^S with non-negative components. For a finite set T, $\Delta(T)$ denotes the set of all probability distributions on T.

As a non-cooperative bargaining procedure for a game (N, v), we consider the random proposer model. Let p be a function that assigns to every coalition $M \subset N$ a probability distribution $p^M \in \Delta(M)$. We refer to p as the *recognition probability*.

The bargaining rule is as follows. Negotiations take place over a (possibly) infinite number of rounds t $(= 1, 2, \ldots)$. Let N^t $(\subset N)$ be the set of all players who remain in the game in round t. For $t = 1$, we set $N^1 = N$. At the start of each round t, one player $i \in N^t$ is selected as a proposer according to the probability distribution $p^{N^t} \in \Delta(N^t)$. Player i proposes a coalition S with $i \in S \subset N^t$ and a payoff allocation $x^S \in X_+^S$. All other members in S either accept or reject the proposal (S, x^S) sequentially. The order of responders does not affect the result in any critical way. If all responders accept the proposal, then the coalition S forms and all its members quit the game. Thereafter, negotiations proceed to the next round $t + 1$, and the same process is repeated with $N^{t+1} = N^t - S$. Otherwise, negotiations continue in the next round $t + 1$ with $N^{t+1} = N^t$. The game ends when no players remain in the negotiations.

The player payoffs are defined as follows. When a proposal (S, x^S) is agreed in round t, every player $i \in S$ receives $\delta_i^{t-1} x_i^S$, where δ_i $(0 \leqslant \delta_i < 1)$ is the discount factor for future payoffs for player i. When the game does not stop, all

[1] Shaked and Sutton (1984) characterized involuntary unemployment as a perfect equilibrium of a non-cooperative bilateral bargaining model.