Games and Economic Behavior 77 (2013) 21–40

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Admissibility and event-rationality ☆

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ARTICLE INFO

Article history:
Received 4 October 2011
Available online 11 September 2012

JEL classification:
C70
C72

Keywords:
Epistemic game theory
Admissibility
Iterated weak dominance
Common knowledge
Rationality
Completeness

ABSTRACT

We develop an approach to providing epistemic conditions for admissible behavior in games. Instead of using lexicographic beliefs to capture infinitely less likely conjectures, we postulate that players use tie-breaking sets to help decide among strategies that are outcome-equivalent given their conjectures. A player is event-rational if she best responds to a conjecture and uses a list of subsets of the other players' strategies to break ties among outcome-equivalent strategies. Using type spaces to capture interactive beliefs, we show that event-rationality and common belief of event-rationality (RCBER) imply $S^\infty W$, the set of admissible strategies that survive iterated elimination of dominated strategies. By strengthening standard belief to *validated belief*, we show that event-rationality and common validated belief of event-rationality (RCvBER) imply IA, the iterated admissible strategies. We show that in complete, continuous and compact type structures, RCBER and RCvBER are nonempty, hence providing epistemic criteria for $S^\infty W$ and IA.

1. Introduction

As noted by Samuelson (1992) and many others, there is a potential problem in dealing with common knowledge of admissibility in games, which is known as the inclusion–exclusion problem. The reason is that, under the assumptions of probabilistic beliefs and expected utility, a strategy is admissible if and only if it is a best response to a belief with full support. So a natural way of obtaining the prediction of admissible choices is to require that players consider all strategies of their opponents to be possible. But then the prediction of an admissible choice for a player is accompanied by a belief that does not exclude any strategy of the player's opponents from consideration, in particular it does not exclude strategies that are not admissible. So a player cannot be certain that the opponents do not play inadmissible strategies.

Recently, Brandenburger, Friedenberg and Keisler (2008), henceforth BFK, provided a way of dealing with the inclusion–exclusion issue, by using lexicographic expected utility (LEU) and the notion of *assumption* in the place of *certainty*. Roughly speaking, a player with a list of probabilistic beliefs can have a fully supported overall belief while "assuming" certain events that are not equal to the whole state space. BFK show that strategies that survive $m+1$ rounds of iterated elimination of inadmissible strategies are the strategies compatible with Rationality and mth-order Assumption of Rationality (RmAR), for every natural number m. However, the limiting construction as $m \to \infty$, RCAR, is empty in complete and continuous type structures. Therefore, BFK do not provide an epistemic characterization of IA. Keisler and Lee (2011) and Yang (2009) have

☆ We are grateful to seminar participants at Collegio Carlo Alberto, Rochester, Southampton, Stony Brook, Cyprus, the Fall 2009 Midwest International Economics and Economic Theory Meetings, the 2010 Royal Economic Society Conference, the 10th SAET Conference, the 2010 Workshop on Epistemic Game Theory, Stony Brook, the CRETE 2010 in Tinos and the Second Brazilian Workshop of the Game Theory Society. We thank an anonymous referee and especially the editor in charge, for their careful and insightful suggestions.
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0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2012.08.012

recently extended BFK's analysis and obtained nonemptiness of RCAR. The former allows for discontinuous type mappings, and the latter uses a weaker notion of assumption. Perea (2012) shows that common assumption of rationality is always possible in finite structures.

We propose an alternative route. Instead of an LEU-based analysis, we use event-rationality to allow for players to break ties with lists of subsets of opponents' strategies. That is, we use a different notion of rationality: the LEU-based approaches assume that players are lexicographic expected utility maximizers. We assume that players are event-rational. The two notions of rationality are equally capable of reconciling "belief of rationality" with "admissible choice". The difference comes into play in the analysis of interactive beliefs. Interactive beliefs are described by type spaces. In our framework, a type of a player determines her beliefs over the strategies and types of the other players (as in the standard framework) and in addition it determines the tie-breaking list of events that the (event-rational) type uses. As a result, common belief of event-rationality bypasses the inclusion–exclusion issue. In contrast, in an LEU-based analysis a type of a player determines her lexicographic beliefs over the strategies and types of the other players, and the inclusion–exclusion tension is bypassed by the use of "assumption" in the place of certainty. Under our approach, we provide epistemic foundations for both the solution concept proposed by Dekel and Fudenberg (1990) ($S^\infty W$) and iterated admissibility (IA).

We consider finite two-player games in strategic form. The two players are Ann and Bob, denoted by superscripts "a" and "b". In order to provide some intuition about event-rationality, note that if a strategy s^a of Ann's is (expected utility) rational then it is a best response to some probabilistic belief, $\nu \in \Delta(S^b)$, where S^b is the set of Bob's strategies. If s^a is inadmissible and therefore weakly dominated by some (mixed) strategy σ^a, then s^a and σ^a give the same payoff for all strategies of Bob in the support of ν while σ^a is strictly better than s^a for all probability measures with support equal to the complement of the support of ν. Hence, when Ann chooses an admissible strategy, it is as if Ann optimizes given the belief ν, as usual, but when she is completely indifferent between two strategies, she compares their expected utilities with respect to a probability measure with support equal to the complement of the support of ν. We say that Ann "breaks ties" using the event that is the complement of the support of ν.

Event-rationality does not require that Ann breaks ties only with respect to the complement of the support of her belief. Ann can conceivably break ties using any other set, as long as it is outside her current frame of mind, that is, disjoint from the support of ν.[1] Furthermore, Ann need not use a single such tie-breaking set. She may well have many such sets, each providing extra validation for the chosen strategy. We refer to a collection of tie-breaking sets as a tie-breaking list.

The principle behind event-rationality is, therefore, the following: if two strategies are outcome-equivalent given Ann's belief, then Ann has no way of deciding among them within her frame of mind: the two strategies yield the same outcome for whichever strategy of Bob she considers possible. Ann must, therefore, resort to information beyond her frame of mind to make a decision. For instance, she could resort to fully external means, like coin flips. However, Ann would be neglecting information about the two strategies under consideration, namely how they fare against strategies of Bob that are considered impossible by her belief. Event-rationality postulates that Ann does not neglect this information and, at the same time, she does not change what she thinks about Bob's choices.

Turn now to interactive beliefs, captured by type structures. Let T^a and T^b be the sets of types of Ann and Bob. A type $t^a \in T^a$ determines Ann's conjectures over Bob's choices, Ann's beliefs over Bob's types and so on, together with the tie-breaking list. A state for Ann is a strategy–type pair (s^a, t^a) and the beliefs over Bob are given by probability measures over $S^b \times T^b$. A strategy–type pair (s^a, t^a) of Ann's is called event-rational if s^a is optimal given t^a's belief over S^b and breaks ties for all sets in t^a's tie-breaking list. States where event-rationality and common belief of event-rationality obtain are captured as the intersection of infinitely many events: Ann is event-rational, and so is Bob; Ann is certain that Bob is event-rational and Bob is certain that Ann is event-rational. And so on. This yields our RCBER ((Event) Rationality and Common Belief of Event-Rationality) set of states.

Event-rationality captures the idea of choosing a strategy with extra validation, in the sense that a strategy has to be optimal under one's belief and in addition it has to pass a series of validating tie-breaking tests. We also introduce the idea of extra validation of a belief. Consider a type t^a that believes that an event $E \in S^b \times T^b$ is true, and is associated with a list ℓ of subsets of S^b. The belief on the event E will be validated by the list ℓ if there is an element of the list, say $E^b \in \ell$, that is equal to the projection of E on S^b.

States where event-rationality and common validated belief of event-rationality obtain are again captured as the intersection of infinitely many events: Ann and Bob are event-rational. Ann has a validated belief that Bob is event-rational and Bob has a validated belief that Ann is event-rational. And so on. This yields our RCvBER ((Event) Rationality and Common validated Belief of Event-Rationality) set of states.

Our results are as follows. We show that in a complete structure, RCBER produces the set of strategies that survive one round of elimination of inadmissible strategies followed by iterated elimination of strongly dominated strategies ($S^\infty W$), whereas RCvBER produces the set of iterated admissible strategies (IA). We then show that strategies played under RCvBER constitute a self-admissible set (SAS), but the converse is not necessarily true. Because BFK have shown that every SAS is the implication of RCAR in some type structure, the RCvBER construction is more restrictive than the RCAR construction of BFK.

[1] But note that, for the purpose of breaking ties, it suffices to consider only subsets of Bob's strategies. In particular, when we introduce the formal model of interactive beliefs, it is without loss of generality to assume that Ann uses only lists of Bob's strategies to break ties, because lists that include the types of Bob only matter for breaking ties through the strategies of Bob that they are related to.