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# Unawareness, beliefs, and speculative trade ☆

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ABSTRACT

We define a generalized state-space model with interactive unawareness and probabilistic beliefs. Such models are desirable for potential applications of asymmetric unawareness. Applying our unawareness belief structures, we show that the common prior assumption is too weak to rule out speculative trade in all states. Yet, we prove a generalized "No-speculative-trade" theorem according to which there cannot be common certainty of strict preference to trade. Moreover, we prove a generalization of the "No-agreeing-to-disagree" theorem.



## 1. Introduction

Unawareness is probably the most common and most important kind of ignorance. Business people invest most of their time not in updating prior beliefs and crossing out states of the world that they previously assumed to be possible. Rather, their efforts are mostly aimed at exploring unmapped terrain, trying to figure out business opportunities that they could not even have spelled out before. More broadly, every book we read, every new acquaintance we make, expands our horizon and our language, by fusing it with the horizons of those we encounter, turning the world more intelligible and more meaningful to us than it was before (Gadamer, 1960).

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With this in mind, we should not be surprised that the standard state-spaces aimed at modeling knowledge or certainty are not adequate for capturing unawareness (Dekel et al., 1998). Indeed, more elaborate models are needed (Fagin and Halpern, 1988; Modica and Rustichini, 1994, 1999; Halpern, 2001). In all of these models, the horizon of propositions the individual has in her disposition to talk about the world is always a genuine part of the description of the state of affairs.

Things become even more intricate when several players are involved. Each player may not only have different languages, but may also form a belief on the extent to which other players are aware of the issues that she herself has in mind. Even more complex, the player may be uncertain as to the sub-language that each other player attributes to her or to others; and so on.

Heifetz et al. (2006) showed how an unawareness structure consisting of a *lattice of spaces* is adequate for modeling mutual unawareness. Every space in the lattice captures one particular horizon of meanings or propositions. Higher spaces capture wider horizons, in which states correspond to situations described by a richer vocabulary. The join of several spaces – the lowest space at least as high as every one of them – corresponds to the fusion of the horizons of meanings expressible in these spaces.

In a companion work (Heifetz et al., 2008), we showed the precise sense in which such unawareness structures are adequate and general enough for modeling mutual unawareness. We put forward an axiom system, which extends to the multi-player case a variant of the axiom system of Modica and Rustichini (1999). We then showed how the collections of all maximally-consistent sets of formulas in our system form a canonical unawareness structure.[1] In a parallel work, Halpern and Rêgo (2008) devised another sound and complete axiomatization for our class of unawareness structures.[2]

In this paper we extend unawareness structures so as to encompass probabilistic beliefs (Section 2) rather than knowledge or ignorance. These unawareness belief structures can be viewed as the "unawareness version" of type spaces in the sense of Harsanyi (1967/68) and Mertens and Zamir (1985). The definition of types (Definition 1), and the way beliefs relate across different spaces of the lattice, is a non-trivial modification of the coherence conditions for knowledge operators in unawareness structures, as formulated in Heifetz et al. (2006). We show that we obtain all properties of unawareness suggested in the literature.

Having structures with both unawareness and probabilistic beliefs raises the question about the differences between probability zero events and events that an agent is unaware of. At an epistemic level, unawareness has very different properties than probability zero belief. For instance, one property that is satisfied by unawareness is symmetry (see Proposition 5). An agent is unaware of an event if and only if she is unaware of its negation. Clearly, such a property cannot be satisfied by probability zero belief because if an agent assigns probability zero to an event, then she must assign probability one to its complement. Schipper (forthcoming) shows that this feature captures also behavioral differences between unawareness and probability zero belief. Let's say a decision maker chooses among different contracts for buying a firm. The seconds contract may differ from a first contract only in a consequence for an event $E$ that is disadvantageous to the buyer. If the decision maker is indifferent between both contracts, then this is consistent with $E$ being Savage null. Yet, if the decision maker is also indifferent between the first and a third contract that differs from the first only in assigning this disadvantageous consequence to the negation of the event $E$ instead the event $E$ itself, then this behavior is inconsistent with the negation of the event $E$ or the event $E$ itself being Savage null. The decision maker behaves as if both the event $E$ and its negation are Savage null, which is impossible but consistent with unawareness of the $E$ and of its negation. Thus, when the primitives of a decision model are fixed, unawareness has behavioral implications distinct from zero probability.[3]

In Section 3, we present as an economic application of unawareness belief structures an analysis of speculative trade under unawareness. We start by defining the notion of a common prior in unawareness belief structures. Conceptually, a prior of a player is a convex combination of (the beliefs of) her types (see e.g. Samet, 1998). If the priors of the different players coincide, we have a common prior. A prior of a player induces a prior on each particular space in the lattice, and if the prior is common to the players, the induced prior on each particular space is common as well.

What are the implications of the existence of a common prior? First, we extend an example from Heifetz et al. (2006) and show that *speculative trade* is compatible with the existence of a common prior (Section 1.1). This need not be surprising if one views unawareness as a particular kind of "delusion", since we know that with deluded beliefs, speculative trade is possible even with a common prior (Geanakoplos, 1989). Nevertheless, we show that a positive common prior is *not compatible* with common certainty of *strict preference* to carry out speculative trade. That is, even though types with limited awareness are, in a particular sense, deluded, a common prior precludes the possibility of common certainty of the event that based on private information players are willing to engage in a zero-sum bet with *strictly positive* subjective gains to everybody. This is so because unaware types are "deluded" only concerning aspects of the world outside their vocabulary,

[1] Each space in the lattice of this canonical unawareness structure consists of the maximally consistent sets of formulas in a sub-language generated by a subset of the atomic propositions.

[2] The precise connection between Fagin and Halpern (1988), Modica and Rustichini (1999), Halpern (2001), and Heifetz et al. (2006) is understood from Halpern and Rêgo (2008) and Heifetz et al. (2008). The connection between Heifetz et al. (2006, 2008) and Galanis (forthcoming) is explored in Galanis (2011a). The connection between Li (2009) and Fagin and Halpern (1988) is explored in Heinsalu (forthcoming). The connections with the models of Ewerhart (2001) and Feinberg (2009) are yet to be explored.

[3] Li (2008) studies further the distinction between unawareness of an event and assigning zero probability to it from a decision-theoretic perspective. She distinguishes between pure unawareness and partial unawareness, under which the subjective point of view of the decision maker corresponds to some 'default' actual state. She shows that both kinds of unawareness may lead to different behavior than under standard probabilistic beliefs.