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Bribing in first-price auctions

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ABSTRACT

I study a symmetric 2-bidder IPV first-price auction prior to which one bidder can offer his rival a bribe in exchange for the latter's abstention. I focus on pure and undominated strategies, and on *continuous monotonic* equilibria—equilibria in which the bribing function is continuous and nondecreasing. When types are distributed continuously on the unit interval, such an equilibrium, if it at all exists, is necessarily trivial—its bribing function is identically zero. I provide a sufficient condition for its existence and sufficient conditions for its nonexistence. When the minimum type is strictly positive, a non-trivial equilibrium may exist, but it must be pooling. I provide a sufficient condition for the existence of such an equilibrium. When types are distributed continuously on the unit interval and dominated strategies are allowed, a non-trivial non-pooling equilibrium exists, at least under the uniform prior.

1. Introduction

Bidder collusion is a well-documented problem.[1] The theoretical literature typically models collusion as a pre-auction one-shot interaction among the cartel members. More specifically, collusion is typically modeled either as a revelation game played by the cartel members, in many cases with the aid of an incentiveless third party, or as a "knockout auction"—an auction that the cartel members run among themselves, for the right to participate in the real auction.[2]

This approach suffers from two drawbacks. First of all, it models collusion as a *static* affair. As such, it misses the important issue of *signaling*, which presents itself in *sequential* interactions: during the formation of the collusive agreement, the bidders' moves are indirect signals of their private information, signals that give rise to adverse selection. The second (and related) drawback is that bidders' negotiation seems to be more naturally modeled as simple bargaining protocols, and not, for example, as abstract revelation mechanisms.

Both drawbacks were addressed by Esö and Schummer (2004, henceforth ES), who studied the following extensive form game: two players are about to attend a symmetric second-price auction with independent and private values (henceforth IPV), and prior to the auction one fixed player has an opportunity to offer the other player a bribe in exchange for the latter's abstention. If the bribe is accepted then the briber becomes the sole participant in the auction and obtains the good for free (the reserve price is zero), while if it is rejected then the pre-auction stage ends and both players go on to compete in the auction noncooperatively.

When studying collusion in simple sequential mechanisms, the ES game is the right place to begin—the second-price format is one of the best-known and most-studied auction formats, and the "take-it-or-leave-it" (henceforth TIOLI) protocol is the simplest bargaining protocol. However, one would like to take the analysis further: examination of alternative models will shed light on the driving forces behind the ES results, and will help clarify how these results depend on the specific

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[1] See, for example, Baldwin et al. (1997), Cassady (1967), and Porter and Zona (1993).

[2] Leading examples include Graham and Marshall (1987), Mailath and Zemsky (1991), Marshall and Marx (2007), and McAfee and McMillan (1992).

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details of the ES game. This leads me to study the same game as ES with one exception—instead of second-price, I consider a first-price auction. As we shall see, there are dramatic differences in the predictions of the two models.

The rest of the paper is organized as follows. Section 1.1 outlines the results and Section 1.2 overviews the related literature. Section 2 lays down the formal model, Section 3 contains the results, Section 4 concludes, and Appendix A collects proofs which are omitted from the text.

1.1. Outline of the results

ES study the second-price game under the assumption that the players employ pure strategies and, moreover, they employ their weakly dominant auction-strategies and bid truthfully in all auctions.[3] Assuming a differentiable, strictly increasing, log-concave distribution of valuations, they prove that this game has a unique perfect Bayesian equilibrium (PBE) that involves bribes and in which the bribing function is continuous[4]; moreover, they derive the closed-form description of this equilibrium. The bribing function of this equilibrium—henceforth, the *ES equilibrium*—starts at the origin, it is strictly increasing up to a certain point, and then it becomes flat.[5]

I also take PBE to be the solution concept, and, similarly to ES, I restrict my attention to pure strategies. In most of the paper I assume that the players do not submit bids strictly above their valuation in the auction, an assumption which is weaker than the restriction to undominated-bidding strategies. For simplicity I will abuse language a little and simply refer to it as "undominated bidding."[6]

Under the assumptions of pure strategies and undominated bidding, the following key point obtains: *if the equilibrium bribing function is continuous and nondecreasing on some interval, then it is constant on that interval.* This implies that when the minimal type is zero, any equilibrium in which the bribing function is continuous and nondecreasing (a *continuous monotonic equilibrium*) is *trivial*—the equilibrium bribing function is identically zero. I obtain (in Theorem 2) a sufficient condition for the existence of a trivial equilibrium. I also obtain (in Theorems 3 and 4) sufficient conditions for its nonexistence.[7] These results are for the case where types are distributed continuously on the unit interval.

I also consider a more general type space, an arbitrary compact interval in $\mathbb{R}_+$. The gist of the aforementioned key point—that a continuous nondecreasing equilibrium bribing function must be constant—still has a bite, but this bite is limited. Now, though all briber types must offer the same bribe, this common offer need not be zero. Continuity and monotonicity still imply pooling, but not necessarily triviality.

The driving force behind the aforementioned results stems from the informational link between the bribing stage and the bidding stage: information which is signaled through the bribe influences bidding in the auction that follows the rejection of that bribe. Under the assumptions of pure strategies and undominated bidding, this influence precludes incentive provision. In particular, the only possibility for a continuous and monotonic equilibrium to exist in this case is that it be completely pooling. In this regard, the ES second-price game is special: it is a special feature of the second-price format that bidding in the auction is independent of any information that is inferred from the pre-auction activity.

The aforementioned informational separation in the ES game, which is due to the fact that ES assume truthful bidding in the auction, has two significant implications. First, the bribing function in every equilibrium of the second-price game must be nondecreasing; second, every bribery-involving equilibrium of the second-price game leads to inefficiency with a positive probability. None of these implications is in place under other bidding behaviors. Though my focus is on the first-price game, I briefly revisit the second-price game of ES, without imposing truthful bidding. I construct an equilibrium of this game in which the bribing function is nonmonotonic and under which the allocation of the good is ex post efficient.

After having dealt with general type intervals and having revisited the second-price game, I go back to the first-price game in which types are distributed on the unit interval. This time, however, I do not impose undominated bidding. This parallels the relaxation of truthful bidding in the second-price game. Under the assumption of a uniform type distribution, I construct an equilibrium which, qualitatively speaking, looks like the ES equilibrium of the second-price game: the bribing function starts at the origin, it is strictly increasing at first, and then it becomes flat. Thus, under both the first- and second-price formats the possibility of dominated behavior at the auction stage brings an improvement: in the former case it makes non-trivial equilibrium possible, and in the latter case it makes efficiency possible.

The focus in the paper is on continuous monotonic equilibria. Though continuity is certainly not pathological, it may seem a little restrictive, because discontinuous equilibria—e.g., equilibria involving step functions—sometimes arise naturally in games with adverse selection and a continuum of types.[8] I therefore examine the relaxation of continuity to *restricted discontinuity*. An equilibrium is *restrictedly discontinuous* if its bribing function has at most finitely many discontinuity points. Even under the weaker notion of restricted discontinuity, equilibrium nonexistence raises its ugly head: given a certain

[3] The players face only one auction, but it can be played following different bribing-stage histories. Auctions that follow different bribing-stage histories are, formally speaking, different continuation games.

[4] When player 1 makes the first move in the game and offers a bribe, this bribe is a function of his valuation (type), hence his behavior is given by a *bribing function*.

[5] The bribing function's exact shape depends on the bidders' type distribution. For example, in the case of the uniform distribution, it is piecewise linear.

[6] I relax this assumption in Section 3.4 (and only there).

[7] This nonexistence is under the pure-strategies assumption.

[8] See, for example, Leininger et al. (1989).