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Robustness of intermediate agreements and bargaining solutions

Nejat Anbarci, Ching-jen Sun *

School of Accounting, Economics and Finance, Deakin University, 70 Elgar Road, Burwood, VIC 3125, Australia

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ABSTRACT

Most real-life bargaining is resolved gradually. During this process parties reach intermediate agreements. These intermediate agreements serve as disagreement points in subsequent rounds. We identify robustness criteria which are satisfied by three prominent bargaining solutions, the Nash, Proportional (and as a special case to the Egalitarian solution) and Discrete Raiffa solutions. We show that the "robustness of intermediate agreements" plus additional well-known and plausible axioms, provide novel axiomatizations of the above-mentioned solutions. Hence, we provide a unified framework for comparing these solutions' bargaining theories.

1. Introduction

Nash's bargaining problem is a pair (S, d), where $S \subset \mathbb{R}^n$ is a convex and compact utility possibility set and d is the disagreement point, the utility allocation that results if no agreement is reached by the parties. A bargaining solution f associates each problem (S, d) with a unique point in S. Since Nash's (1950) seminal solution and axioms, various other solutions and axioms have been proposed.

One prominent axiom is the Step-by-Step Negotiation (SSN) axiom (Kalai, 1977). SSN requires that the bargaining outcome be invariant under decomposition of the bargaining process into stages: if parties know that they will face two nested sets in sequence, first a set S and then a superset T, then the solution outcome of S can function as an intermediate agreement for T. Kalai (1977) emphasized the advantage of SSN as follows:

> This principle is observed in actual negotiations (e.g., Kissinger's step-by-step), and it is attractive since it makes the implementation of a solution easier. It is also attractive because we can view every bargaining situation that we encounter in life as a first step in a sequence of predictable or unpredictable bargaining situations that may still arise. Thus, the outcome of the current bargaining situation will be the threat point for the future ones.

Indeed, most real-life bargaining is resolved gradually. During this process parties reach intermediate agreements, and these intermediate agreements serve as new disagreement points and pave the way for subsequent negotiations. Cooperative bargaining solutions ignore these dynamics and can therefore yield accurate predictions only if they are robust to the specifications of these dynamics.

* Corresponding author.
E-mail addresses: nejat.anbarci@deakin.edu.au (N. Anbarci), cjsun@deakin.edu.au (C.-j. Sun).

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Given two bargaining problems (S, d) and (T, e) with $d = e$, an intermediate agreement, d', is considered *robust* if moving the disagreement point from d to d' has no effect to the bargaining outcome in either (S, d) or (T, e). As such, SSN does provide a substantial and specific robustness test. When parties face first (S, d) and then (T, e) with $d = e$ and $S \subset T$, SSN then suggests that they can reach an intermediate agreement at $f(S, d)$, as moving the disagreement point from d to $f(S, d)$ has no effect to the bargaining outcome in (T, e).

In this paper we consider alternatives to SSN's robustness test by taking into account additional facts about the relationship between S and T.

1. S is included in T *and* $f(S, d)$ is not required to be an intermediate agreement.
2. Requirement 1 *and* S and T have the same *ideal payoffs* — i.e., each party's highest payoff subject to the other party receiving her disagreement payoff is the same in both S and T.
3. Requirement 1 *and* the parties expect to receive the same relative gains in S and T.

Thus, in our robustness tests, certain points will serve as intermediate agreements under different circumstances: In Test 1 above, whenever the sets S and T are nested, but any such intermediate agreement is not necessarily the solution outcome of S; in Test 2, whenever S and T are nested and share the same ideal point; in Test 3, whenever S and T are nested and the parties expect to have the same relative gains.

The above robustness tests will be termed "Robustness of Intermediate Agreements" (RIA) axioms. Each of these RIA axioms — when combined with some other well-known and plausible axioms — will lead to the n-person axiomatizations of the Nash, Proportional and Discrete Raiffa solutions. In that sense we provide a unified framework to obtain general (n-person) characterizations of these solutions using different types of sets of intermediate agreements and thereby compare these solutions' bargaining theories in that regard.

Our results can be briefly summarized as follows:

(1) The Nash solution is characterized by Midpoint Domination (MD), an RIA axiom, and Continuity (CONT).
(2) The Proportional solutions (and as a special case to the Egalitarian solution) are characterized by an RIA axiom, Translation Invariance (TI), CONT, Weak Pareto Optimality (WPO) and Strong Individual Rationality (SIR).
(3) The Discrete Raiffa solution is characterized by MD, an RIA axiom, and Independence of Non-Midpoint Dominating Alternatives (INMD).

2. Relevant literature

The significance and fundamental role of bargaining were recognized as early as 1881 by Edgeworth (1881), but for a very long time it was deemed to lack a clear solution. Nash (1950) provided the first axiomatic derivation of a bargaining solution, characterized by four axioms — namely WPO, Symmetry (SYM), Scale Invariance (SI), and Independence of Irrelevant Alternatives (IIA). Raiffa (1953) soon proposed another solution which essentially described a discrete bargaining process, but did not provide an axiomatic characterization of his solution. Kalai and Smorodinsky (1975) characterized a new solution which, like the Discrete Raiffa solution, emphasized the parties' ideal payoffs. Initially, all characterizations employed an *independence* or *monotonicity* axiom pertaining to changes in the feasible set (pioneered by Nash, 1950, and Kalai and Smorodinsky, 1975, respectively).

The second generation characterizations then shifted the focus to *changes in the disagreement payoffs*, as well as to considerations of *uncertain disagreement points* — the latter have been first considered by Chun and Thomson (1990). In the literature of arbitration games, properties — such as convexity — of the disagreement point sets played an explicit role; e.g., Tijs and Jansen (1982). In cooperative bargaining, changes in disagreement payoffs have been first considered by Peters (1986) and Thomson (1987).

The axioms used in these two generations, however, typically did not refer to any bargaining process. Later, MD (Sobel, 1981) and Step-by-Step Negotiation (SSN) (Kalai, 1977) utilized a bargaining process by reaching intermediate agreements that help eliminate the most lop-sided and/or inefficient portions of a utility possibility set S, which at least one of the parties would find undesirable. Moulin (1983) used MD to characterize the Nash solution, and Kalai (1977) used SSN to characterize the Proportional solutions. Rachmilevitch (2012) recently used an axiom, 'Interim Improvement', which refers to a bargaining process, to characterize the Proportional solutions.

There has been a recent revival of interest in Raiffa's work. Livne (1989), Peters (1986), and Peters and van Damme (1991) provided characterizations of a "continuous" version of the Raiffa solution; based on disagreement point axioms, Peters (1986) also provided a unified framework characterizing different solutions. More recently, Trockel (2009) provided an axiomatic characterization of the Discrete Raiffa solution, in which he uses SYM, WPO, SI as well as an axiom that repeatedly uses the other three axioms to improve the disagreement point iteratively. Dishkin et al. (2011) — using the concept of interim agreements — came up with an axiomatic characterization of a family of stepwise bargaining solutions that connects the Discrete Raiffa solution and the Continuous Raiffa solution.