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Elasto-plastic constitutive model for geotechnical materials with strain-softening behaviour

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Abstract

An elasto-plastic model originally developed at Tsinghua University (Tsinghua elasto-plastic model) for soils reflects the observed non-linearity and dilatancy of soils in laboratory tests. The model incorporates a yield criterion, an associated flow rule and a work-hardening law, which can be determined by laboratory tests. This paper presents a modified Tsinghua elasto-plastic constitutive model, which incorporates a function of plastic work as the hardening parameter in the original Tsinghua elasto-plastic model. The modified model has 12 parameters, which are easily determined using conventional triaxial shearing and isotropic compression tests. It can be used to describe strain hardening followed by strain-softening behaviour and to simulate dilatancy during shear application. The modified model was used to determine the relationship between load and settlement for a shallow footing founded on dense Yongding River sand. The results of the numerical analyses were in agreement with the results of hydraulic gradient laboratory tests. The results showed in a preliminary way that the modified model developed in this study was effective for simulating the strain-softening boundary-value problems common in geotechnical engineering.

Keywords: Strain-softening behaviour; Stress–strain relations; Elasto-plastic model; Finite element method; Soil mechanics; Shallow foundation

1. Introduction

Strain-softening behaviour is commonly observed in the testing of geotechnical materials. As such, using stress–strain relationships without incorporating strain-softening behaviour into model where it is warranted may give erroneous results. Over the past two decades, numerous models that account for strain-softening behaviour have been developed (Lade and Duncan, 1975; Dragon and Mroz, 1979; Bažant et al., 1984; Resende and Martin, 1984; Lemaitve, 1985; Kachanov, 1986; Frantziskonis and Desai, 1987a–c; Adachi and Oka, 1995; Yoshida et al., 1995). Strain softening caused by damage or localization can be described very well using damage models (Resende and Martin, 1984; Lemaitve, 1985; Kachanov, 1986; Frantziskonis and Desai, 1987a–c; Matsushima et al., 2002; Al-Shayea et al., 2003; Hu and Pu, 2003). In other cases, strain softening is not caused by damage but by dilation or a decrease

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in the confining pressure. In this situation, damage models cannot be used to explain the mechanism of strain softening.

An elasto-plastic model developed at Tsinghua University (Tsinghua elasto-plastic model) (Huang et al., 1981; Pu et al., 1985) for soils reflects the observed non-linearity and dilatancy of soils in laboratory tests. The model incorporated a yield criterion, an associated flow rule and a work-hardening law. The yield function and the work-hardening law of the Tsinghua elasto-plastic model can be determined using common laboratory tests. In this model, an appropriate work-hardening law, which is a function of plastic volume strain (ε_v^p) and plastic shear strain ($\bar{\varepsilon}^p$), was selected so that the yield surface coincided with the plastic potential surface. A method of determining the hardening parameter was proposed by Sun (1982). A three-dimensional parameter (Li and Pu, 1987) enables the stress–strain model to be used for general three-dimensional conditions. Results of true triaxial tests, plane–shear tests and model pressuremeter tests were analysed using this model. The stress–strain characteristics observed in these tests were predicted with good accuracy (Li, 1985).

Since the hardening parameter is a function of plastic volume strain (ε_v^p) and plastic shear strain ($\bar{\varepsilon}^p$), the hardening parameter of the Tsinghua elasto-plastic model is difficult to express if soil exhibits strain-softening behaviour. In contrast, plastic work is a physical parameter that reflects plastic strain. The value of plastic work monotonically increases whether soil stresses are in a state of strain hardening or in a state of strain softening. Therefore, strain-softening behaviour can be more easily expressed using a function of plastic work as the hardening parameter.

2. Tsinghua elasto-plastic model

The Tsinghua elasto-plastic model was developed by Huang et al. (1981) and Li (1985). Unlike other elasto-plastic models for soil, the plastic potential surface of the Tsinghua elasto-plastic model was determined directly from experimental data without any assumption about the shape of plastic potential surface. According to Drucker's postulation (Drucker, 1959), the yield function (f) was identical to the plastic potential function (g) by selecting a proper hardening parameter.

In the Tsinghua elasto-plastic model, the total strain increment ($d\varepsilon_{ij}$) includes two components: elastic strain increments ($d\varepsilon_{ij}{}^e$) and plastic strain increments ($d\varepsilon_{ij}{}^p$).

2.1. Elastic parameters

The elastic strain increments can be calculated using Hooke's law. The elastic bulk modulus (K) and shear modulus (G) in the model are determined from the unloading–reloading curves of isotropic compression tests and conventional triaxial shearing tests. They are expressed as follows:

$$K = k_p p, \tag{1}$$

$$G = k_g p_a \left(\frac{\sigma_3}{p_a}\right)^{n_g}, \tag{2}$$

where $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$ is the octahedral normal stress; σ_1, σ_2 and σ_3 are the three principal stresses; k_p, k_g and n_g are material constants; p_a is the atmospheric pressure in the same units as stress σ_3. k_p can be determined using the least-squares regression method based on the slope of the unloading–reloading curves of isotropic compression tests. k_g and n_g are also determined using the least-squares regression method after linearization of Eq. (2) based on the unloading–reloading curves of conventional triaxial shearing tests.

2.2. Yield function

In the octahedral stress plane, the direction of plastic strain increments is drawn at every stress point on the stress path of conventional triaxial shearing tests. Similar to drawing a seepage flow net, the plastic vector "flow lines" and equi-plastic potential lines, which are always perpendicular to each other at any point, can be drawn. According to Drucker's postulation, the yield function is identical to the plastic potential function. Fig. 1 shows the

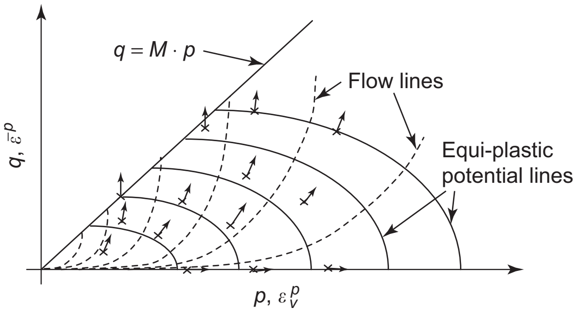

Fig. 1. Yield loci of Chengde sand.