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Influence functions, followers and command games

Michel Grabisch *, Agnieszka Rusinowska

Université Paris I Panthéon-Sorbonne, Centre d'Economie de la Sorbonne, 106–112 Bd de l'Hôpital, 75013 Paris, France

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ABSTRACT

We study and compare two frameworks: a model of influence, and command games. In the influence model, in which players are to make a certain acceptance/rejection decision, due to influence of other players, the decision of a player may be different from his inclination. We study a relation between two central concepts of this model: influence function, and follower function. We deliver sufficient and necessary conditions for a function to be a follower function, and we describe the structure of the set of all influence functions that lead to a given follower function. In the command structure introduced by Hu and Shapley, for each player a simple game called the command game is built. One of the central concepts of this model is the concept of command function. We deliver sufficient and necessary conditions for a function to be a command function, and describe the minimal sets generating a normal command game. We also study the relation between command games and influence functions. A sufficient and necessary condition for the equivalence between an influence function and a normal command game is delivered.

1. Introduction

Success of every activity in life depends to a high degree on how well one can *influence* others. The influence ability helps when creating a favorable impression on others about ourselves and our achievements, e.g., when negotiating for grades, having a job interview, implementing ideas in an organization, or making crucial economic and political decisions.

During the last three decades, the political economic literature has offered many theoretical and empirical studies of political influence and power in groups (see e.g. van Winden, 2004 for a short survey). Although many works on noncooperative models have characterized the field of political economy, a cooperative approach to model influence and power in groups has been applied as well and it is still of great interest. Since influence is present both in noncooperative and cooperative environments, it should be studied by using tools of both noncooperative and cooperative game theory. These two approaches are complementary to each other when studying influence, and hence none of them should be ignored, but they should be rather compared to each other. Applying both the noncooperative and cooperative approaches can give a real picture of what is going on in the world based on interaction between people, and can better explain the influence phenomenon. Starting with a lack of cooperation, one might be better off by switching to a cooperative attitude.

One of the natural phenomena related to influence and interaction among agents is obviously the concept of leadership. According to power and influence theory of leadership, this concept is based on the form of relationships between people rather than on the abilities of a single person. In DeMarzo (1992), the set of outcomes sustainable by a leader with the power to make suggestions in games is examined. These suggestions are important even if players can communicate and form coalitions. The author considers both finite-horizon games and infinite-horizon two-player repeated

* Corresponding author. Fax: +33 144 078 301.
E-mail addresses: michel.grabisch@univ-paris1.fr (M. Grabisch), agnieszka.rusinowska@univ-paris1.fr (A. Rusinowska).

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games. In order to understand how power, influence, and leadership are related to various facets of organizational life at the individual, group and macro levels, network formation is usually applied. The literature on network formation has been growing up very fast for the last decade (see, e.g., Jackson and Wolinsky, 1996; Jackson and Van den Nouweland, 2005; Bloch and Dutta, 2009). Many noncooperative models of network formation have been investigated (see, e.g., Bala and Goyal, 2000), but also a cooperative approach to networks has been applied (see, e.g., Jackson, 2005). In Koller and Milch (2003) the authors propose a graphical representation for noncooperative games called multi-agent influence diagrams (MAIDs), which represent decision problems involving multiple agents. One also needs to mention the use of social networks in studying influence, because, as mentioned in Hojman and Szeidl (2006), individual decisions and strategic interaction are both embedded in a social network. In Lopez-Pintado (2008), for instance, the author stresses the fact that decisions of individuals are often influenced by the decisions of other individuals. She considers a network of interacting agents whose actions are determined by the actions of their neighbors, according to a diffusion rule.

Another approach to modeling players' interactions by the use of a social network has been proposed in Hoede and Bakker (1982). In this model, each player is assumed to have an *inclination* to say either YES or NO which, due to influence by other voters, may be different from the *decision* of the player. The influence model introduced in Hoede and Bakker (1982) is broader than voting models, because in the influence model the analysis begins not in voting itself, but 'one step earlier', that is, in the framework of original inclinations of voters. *Influence* by other players in this model means that a player's vote (decision) is different from his original inclination. Formally, the influence is expressed by an *influence function*, which assigns to each inclination vector (i.e., a vector describing the inclinations of all players) a decision vector (i.e., a vector indicating the decisions of the players). The approach based on influence functions is not cooperative in essence, since it does not involve any notion of game in the cooperative sense. Rather, since influence is a dynamical concept which leads to some equilibrium (final decision after several steps of influence), it has a noncooperative flavor. One of the tools that describes the influence function is the concept of a follower of a given coalition under a given influence function, that is, a voter who always follows the inclination of the coalition in question. Formally, a *follower function*, which assigns to each coalition the set of its followers, is defined. This influence model is studied e.g. in Grabisch and Rusinowska (2010) where, in particular, we introduce weighted influence indices, and consider different influence functions.

Since influence among players is one of the natural phenomena that may appear in particular in voting situations, modeling interaction among voters via voting games has been also presented in the literature. Among concepts related to this topic it is worth mentioning the notion of *influence relation* in simple games, which was introduced fifty years ago in Isbell (1958), to qualitatively compare the a priori influence of voters in a simple game. As defined in Isbell (1958), in a simple game, where players can vote either YES or NO, voter k is said to be at least as influential as voter j, if whenever j can transform a losing coalition into a majority by joining it, voter k can achieve the same *ceteris paribus*. Very recently, in Tchantcho et al. (2008) the influence relation has been extended to voting games with abstention. The concept of interaction among players in a cooperative game is also studied, for instance, in Grabisch and Roubens (1999), where players in a coalition are said to exhibit a positive (negative) interaction when the worth of the coalition is greater (smaller) than the sum of the individual worths.

Another interesting model related to the topic in question has been recently presented in Hu and Shapley (2003a, 2003b), where the command structure of Shapley (1994) is applied to model players' interaction relations by simple games. This approach is typically of cooperative nature. For each player, boss sets and approval sets are introduced, and based on these sets, a simple game called the *command game* for a player is built. Given a set of command games, the *command function* is defined, which assigns to each coalition the set of all players that are 'commandable' by that coalition. In Grabisch and Rusinowska (2009) we compare the framework of command games with the influence model. In particular, we define several influence functions which capture the command structure. These functions are compatible with the command games, in the sense that each commandable player for a coalition in the command game is a follower of the coalition under the command influence function. Some of the presented influence functions are equivalent to the command games: An influence function and a command game are said to be *equivalent* if the follower function of this influence function is identical to the command function in this command game. For some influence functions we define the equivalent command games. Moreover, we show that not for all influence functions the compatible command games exist.

Although both cooperative and noncooperative approaches to power and influence have been presented in the economic literature, research on the relations between these two approaches has not been conducted frequently so far. The exception in the field of voting power can be found, e.g., in Laruelle and Valenciano (2008), where a noncooperative interpretation of power indices has been provided. The authors model noncooperative bargaining processes and show how the power indices (i.e. the Shapley–Shubik index) can be interpreted as measures of bargaining power that appear as limit cases (see also Laruelle and Valenciano, 2009 for cooperative bargaining foundation of this index).

Our aim is to model influence in a broad sense, by studying both the noncooperative and cooperative approaches to influence in groups and, in particular, by determining *links between these two approaches*. An advantage of our influence framework is its generality which allows to cover many different situations. This is due to the influence function which can be defined arbitrarily. On the one hand, our model covers noncooperative aspects, when an individual tries to change the opinion of other agents and makes them decide differently from their preliminary plans in order to end up in the individual's preferable outcome. On the other hand, the model has also cooperative features with individuals who form coalitions and try to 'win the game' by cooperation.