Games and Economic Behavior 72 (2011) 526–543

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

One-dimensional bargaining

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ARTICLE INFO

Article history:
Received 29 September 2008
Available online 28 October 2010

JEL classification:
C78

Keywords:
Bargaining
Subgame perfect equilibrium
Acceptance rules

ABSTRACT

We study a model of multilateral bargaining over social outcomes represented by the points in the unit interval. The acceptance or rejection of a proposal is determined by an acceptance rule represented by the collection of decisive coalitions. The focus of the paper is on the asymptotic behavior of subgame perfect equilibria in stationary strategies as the players become infinitely patient. We show that, along any sequence of stationary subgame perfect equilibria the social acceptance set collapses to a point. This point, called the limit of bargaining equilibria, is independent of the sequence of equilibria and is uniquely determined by the set of players, the utility functions, the recognition probabilities, and the acceptance rule. The central result of the paper is a characterization of the limit of bargaining equilibria as a unique zero of the characteristic equation.

1. Introduction

This paper analyzes a model of multilateral bargaining where the available alternatives are represented by the points in the unit interval. Thus the alternative might represent a level of taxation, a location of a facility, or an index of an ideological content of a policy (left vs. right).

Bargaining proceeds as follows. At the beginning of each period, nature randomly selects one of the players as a proposer. The probability for a player to become a proposer, the so-called recognition probability, is assumed to be the same in each period. The player chosen by nature puts forward a proposal specifying one alternative. All players then react to the proposal. Each player can either reject or accept the proposal. Whether the proposal passes or fails is then determined by the acceptance rule, as represented by the collection of decisive coalitions. The passing of a proposal requires an approval of it by all the players in some decisive coalition. Examples of acceptance rules are the unanimity acceptance rule when a passing of a proposal requires an approval of it by all the players, and the quota rule, when a fixed number of votes is needed for a passing of a proposal. If the proposal passes, it is implemented and the game ends. In this case each player receives a discounted utility of the implemented alternative. Otherwise, a new period begins.

We consider subgame perfect equilibria in stationary strategies. Stationarity means that a proposal of any player does not depend on the history of play and a player's reaction to a proposal depends only on the proposal itself. The focus of the paper is on the asymptotic behavior of stationary subgame perfect equilibria as the discount factor approaches one.

We prove that subgame perfect equilibria in stationary strategies are *asymptotically unique* in the following sense: Along any sequence of subgame perfect equilibria in stationary strategies the social acceptance set collapses to a point. This point, called the limit of bargaining equilibria, is independent of the sequence of equilibria and is uniquely determined by the set of players, the utility functions, the recognition probabilities, and the acceptance rule. The central result of the paper is a characterization of the limit of bargaining equilibria as a unique zero of the characteristic equation.

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[1] The author acknowledges the financial support of the Dutch Science Foundation (NWO) through a VENI-grant. The author is most thankful to Jean-Jacques Herings and the anonymous referees for their suggestions.

0899-8256/$ – see front matter
doi:10.1016/j.geb.2010.10.007

The results are obtained under rather minimal assumptions. Thus the utility functions are only assumed to be non-negative, single-peaked and concave. Furthermore, we require that the intersection of any two decisive coalitions contain a player with a positive recognition probability. This requirement puts but a very mild restriction on the acceptance rule and the recognition probabilities.

Various results on one-dimensional bargaining have been previously obtained in Banks and Duggan (2000), Cho and Duggan (2003), Kalandrakis (2006), Cardona and Ponsatí (2007), Herings and Predtetchinski (2010), Imai and Salonen (2000), Compte and Jehiel (2010). Banks and Duggan (2000) consider bargaining in a situation where the alternatives are represented by points in a general compact convex set. For the special case where the set of alternatives is one dimensional they establish the existence of stationary subgame perfect equilibria in pure strategies.

The question of uniqueness of stationary subgame perfect equilibria in the one-dimensional bargaining game is addressed in Cho and Duggan (2003), Cardona and Ponsatí (2007), Herings and Predtetchinski (2010). Cho and Duggan (2003) derive the uniqueness of subgame perfect equilibria in pure stationary strategies assuming that the utility functions are quadratic and the acceptance rule is proper and strong. Cardona and Ponsatí (2007) show that stationary subgame perfect equilibria in pure strategies are unique in a game where the proposers rotate in a fixed sequence, provided that each player's utility function is symmetric around the peak and the acceptance rule is a quota rule. Herings and Predtetchinski (2010) establish the uniqueness result in a model where the identity of the proposer follows a general Markov process, assuming tent-shaped utility functions and the unanimity acceptance rule.

In general, a one-dimensional bargaining game can have many stationary subgame perfect equilibria, examples of multiplicity given in Cho and Duggan (2003) and Kalandrakis (2006). In particular, the 7-player game in Kalandrakis (2006) has a continuum of pure strategy stationary subgame perfect equilibria. As is demonstrated in Cho and Duggan (2003), stationary equilibria are nested in the sense that the social acceptance set in one equilibrium forms the subset of the social acceptance set of the other equilibrium. Moreover, Kalandrakis (2006) shows that pure strategy stationary subgame perfect equilibria are locally unique and finite in number for almost all discount factors.

Asymptotic uniqueness of stationary subgame perfect equilibria in the one-dimensional bargaining game has been shown in Cardona and Ponsatí (2007). In this paper not only do we show that equilibria are asymptotically unique, but we also provide a description of the limit by means of a characteristic equation. A characterization of the limit of equilibria is also given in Herings and Predtetchinski (2010), but this result only applies in a game with the unanimity acceptance rule and the tent-shaped utility functions. In the case of time-invariant recognition probabilities the characterization of Herings and Predtetchinski (2010) follows as a corollary to our main result.

Imai and Salonen (2000) introduce the concept of representative Nash bargaining solution for the situation of two-sided bargaining. In a two-sided bargaining problem the alternatives are represented by points in the interval and the players are divided into two groups with diametrically opposite preferences. The authors provide an axiomatization of the representative Nash bargaining solution and the non-cooperative characterization of it as a limit of stationary equilibrium in a game of bargaining when the probability of the breakdown of negotiations vanishes. Unlike Imai and Salonen (2000), we allow for players to have intermediate preferences.

Compte and Jehiel (2010) study the performance of majority rules in bargaining assuming that the proposals at any round of negotiations are drawn by nature randomly from a fixed distribution over the set of alternatives, so the players have no influence on the proposals. They demonstrate that the set of accepted proposals shrinks to a point as the players become infinitely patient, and provide a characterization of the limit proposal. When the set of alternatives is one dimensional they show that the limit proposal is determined by the bliss points of only two players (called by the authors the "decisive" players). For the bargaining with unanimous consent, the decisive players are the players with the lowest and the highest ideal points. Furthermore, the limit proposal coincides with the Nash bargaining solution of the game where only the two decisive players are present. Both Compte and Jehiel (2010) and Banks and Duggan (2000) also find that under the simple majority rule the limit of stationary equilibria coincides with the ideal point of the median player.

Numerous contributions study stationary subgame perfect equilibria in games of multilateral bargaining where the players have to agree upon a division of an amount of money among themselves, in which case the alternatives are represented by points in the simplex. Thus Merlo and Wilson (1995, 1998) give sufficient conditions for the uniqueness of stationary subgame perfect equilibria (in pure strategies) in a model where the identity of the proposer and the amount of money to be divided follow a Markov process and the unanimous approval is needed for a proposal to pass. Eraslan (2002) establishes uniqueness of stationary subgame perfect equilibria (in mixed strategies) in a game with a tree similar to the one described above and the quota acceptance rule. Eraslan and Merlo (2002) characterize stationary subgame perfect equilibria (in mixed strategies) in a model where the amount of money to be divided is stochastic and a general agreement rule is used.

The study of the asymptotic behavior of stationary subgame perfect equilibria in Hart and Mas-Colell (1996), Miyakawa (2006), Laruelle and Valenciano (2009), and Britz et al. (2010) provides a non-cooperative foundation for the asymmetric Nash bargaining solution. It is shown that the limit of stationary subgame perfect equilibria in the n-player game as the probability of the breakdown of negotiations goes to zero converges to the asymmetric Nash bargaining solution, with the weights given by the players' recognition probabilities. Compte and Jehiel (2010) obtain a similar result in a model where the proposals are randomly drawn by nature rather than being chosen by the players. Kultti and Vartiainen (2007) obtain the asymmetric Nash bargaining solution as a limit of the Von Neumann–Morgenstern stable set as the discount factor vanishes. The stable set is defined with respect to a dominance relation where an alternative x dominates an alternative y if some player prefers x even with a one-period delay.