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Leadership games with convex strategy sets ☆

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ABSTRACT

A basic model of commitment is to convert a two-player game in strategic form to a "leadership game" with the same payoffs, where one player, the leader, commits to a strategy, to which the second player always chooses a best reply. This paper studies such leadership games for games with convex strategy sets. We apply them to mixed extensions of finite games, which we analyze completely, including nongeneric games. The main result is that leadership is advantageous in the sense that, as a set, the leader's payoffs in equilibrium are at least as high as his Nash and correlated equilibrium payoffs in the simultaneous game. We also consider leadership games with three or more players, where most conclusions no longer hold.

1. Introduction

The possible advantage of commitment power is a game-theoretic result known to the general public, ever since its popularization by Schelling (1960). Cournot's (1838) duopoly model of quantity competition was modified by von Stackelberg (1934), who demonstrated that a firm with the power to commit to a quantity of production profits from this leadership position. The leader–follower issue has been studied in depth in oligopoly theory as "Stackelberg leadership"; see Friedman (1977), Hamilton and Slutsky (1990) and the correction to that paper by Amir (1995), Shapiro (1989), or Amir and Grilo (1999) for discussions and references.

We define a leadership game as follows (for details see Section 2). Consider a game of $k+1$ players in strategic form. Declare one player as *leader* and let his strategy set be X. The remaining k players are called *followers*. Let the set of their partial strategy profiles (with k strategies) be Y, so that $X \times Y$ is the set of full strategy profiles. The leadership game is the extensive game where the leader chooses x in X, the followers are informed about x and choose simultaneously their strategies as $f(x)$ in Y, and all players receive their payoffs as given by the strategy profile $(x, f(x))$. We only consider subgame perfect equilibria of the leadership game where for any x the followers play among themselves a Nash equilibrium $f(x)$ in the game induced by x, even off the equilibrium path. We call $f(x)$ the *response* of the followers to x, which is simply a best reply in the original game if there is only one follower. (The set of equilibria that are not subgame perfect seems too large to allow any interesting conclusions.)

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Our aim is to analyze completely leadership games for the mixed extension of a bimatrix game, that is, of a finite two-player game in strategic form. Then there is only one follower ($k = 1$). The leader commits to a mixed strategy x in the bimatrix game. The follower's response $f(x)$ is also a mixed strategy. The pair of pure actions is then chosen independently according to x and $f(x)$ with the corresponding bimatrix game payoffs, and the players maximize expected payoffs as normally.

The payoff to the leader in a subgame perfect equilibrium of the leadership game is called a *leader payoff*. His payoff in a Nash equilibrium of the simultaneous game is called a *Nash payoff*. When considering the simultaneous game, we often have to identify the player who becomes leader in the corresponding leadership game; for simplicity of identification, we call this player also "leader" in the simultaneous game.

For the mixed extension of a bimatrix game, our main result (Corollary 8) states that the set of leader payoffs is an interval $[L, H]$ so that $H \geqslant E$ for all Nash payoffs E, and $L \geqslant E$ for at least one Nash payoff E. Furthermore, Theorem 12 states that $H \geqslant C$ for any correlated equilibrium payoff C to the leader. In this sense, the possibility to commit, by changing a simultaneous game to a leadership game, never harms the leader. However, this no longer holds for two or more followers, where leadership can be disadvantageous (see Remark 5).

One motivation to consider commitment to mixed strategies is the "classical view" of mixed strategies (see also Reny and Robson, 2004). This is the view of von Neumann and Morgenstern (1947), who explicitly define the leadership game corresponding to a zero-sum game, first with commitment to pure strategies (p. 100) and then to mixed strategies (p. 149), as a way of introducing the max–min and min–max value of the game. They consider the leader to be a priori at an obvious disadvantage. By the minimax theorem, a player is not harmed even if his opponent learns his optimal mixed strategy. Hence, in two-person zero-sum games, commitment to a mixed strategy does not hurt the leader, in line with Corollary 8. The value of a zero-sum game is its unique leadership and Nash payoff.

Important applications of commitment to mixed strategies are inspection games. They model inspections for arms control treaties, tax auditing, or monitoring traffic violations; for a survey see Avenhaus et al. (2002). With costly inspections, such games typically have unique mixed equilibria, and in the corresponding leadership games, the inspector is a natural leader. As observed by Maschler (1966), commitment helps the leader because the follower, who is inspected, acts legally in an equilibrium of the leadership game, but acts illegally with positive probability in the Nash equilibrium of the simultaneous game.

The central observation about leadership games for mixed extensions of bimatrix games is the following. When the leader commits to his mixed strategy in equilibrium, the follower is typically indifferent between several pure best replies. However, the condition of subgame perfection implies that on the equilibrium path, the follower chooses the reply that gives the *best* possible payoff to the leader; otherwise, the leader could improve his payoff by changing his commitment slightly so that the desired reply is unique (which is possible generically).

For inspection games, this reasoning based on subgame perfection was used by Avenhaus, Okada, and Zamir (1991). Maschler (1966) still postulated a benevolent reaction of the follower when she is indifferent (and called this behavior "pareto-optimal"), or else suggested to look in effect at an ε-equilibrium in which the leader sacrifices an arbitrarily small amount to induce the desired reaction of the follower. A similar observation is known for bargaining games, for example in the iterated offers model of Rubinstein (1982). In a subgame perfect equilibrium of this game, the first player makes the second player indifferent between accepting or rejecting the offer, but the second player nevertheless accepts.

Some of our results apply to more general games than mixed extensions of bimatrix games. In particular, we are indebted to a referee who suggested a short proof of Corollary 8 based on Kakutani's fixed point theorem. Different parts of Corollary 8 hold under assumptions that can be weakened to varying extent. We therefore present these parts separately, as follows.

In Section 2, we give a characterization in Theorem 1 of the lowest leader payoff, using standard assumptions so that Kakutani's fixed point theorem can be applied, for games with any number of followers. Suppose that they always choose their response to give the worst possible payoff to the leader. In other words, the leader maximizes his payoff under the "pessimistic" view that the followers act to his disadvantage. (This pessimistic view is also used to the define a "Stackelberg payoff" to the leader in dynamic games; see Başar and Olsder (1982, Eq. (41) on p. 136, and p. 141).) The resulting payoff function to the leader is typically discontinuous and has no maximum (see also Morgan and Patrone (2006) and references). However, the *supremum* of the "pessimistically" computed payoff to the leader is obtained in a subgame perfect equilibrium of the leadership game, as we show in Theorem 1. Subgame perfection implies that on the equilibrium path, the followers' response is *not* according to the pessimistic assumption but instead yields the supremum payoff to the leader.

In Section 3, we observe in Theorem 2 that the lowest leader payoff is no worse than the lowest Nash payoff. This theorem requires strong assumptions that hold for mixed extensions of bimatrix games (Corollary 3), but not, for example, for mixed extensions of three-player games (Remark 5). Furthermore, the highest leader payoff is obtained when the followers always reply in the best possible way for the leader. It is easy to see that this payoff is at least as high as any Nash payoff. Moreover, if the set of the followers' responses is connected, then the set of leader payoffs is an interval (Proposition 7).

In Section 4, we consider mixed extensions of bimatrix games. We explicitly characterize the lowest and highest leader payoffs and show how to compute them by linear programming. For generic games, they are equal.

In Section 5, we show that the highest leader payoff H is at least as high as any correlated equilibrium payoff to the leader. This is no longer true for the coarse correlated equilibrium due to Moulin and Vial (1978) that involves commitment by both players, which may give a higher payoff than H.