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Note

On multiple-principal multiple-agent models of moral hazard ☆

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ABSTRACT

We provide two examples in a pure moral hazard setting with two principals and two agents. Example 1 shows that a strongly robust equilibrium in simple (direct) mechanisms can no longer be sustained as an equilibrium when a principal can deviate to an indirect communication scheme. Conversely, an equilibrium with one principal offering an indirect mechanism cannot be replicated as an equilibrium in simple mechanisms. Example 2 shows more directly that a payoff profile that can be achieved in equilibrium when one principal offers an indirect mechanism cannot be achieved as an equilibrium profile in simple mechanisms.

1. Introduction

Consider an environment with two principals, two agents, and pure moral hazard. That is, there is complete information over agents' types and effort is non-contractible. In this environment, we define a simple mechanism as one in which a principal privately recommends a specific effort to each agent. A simple mechanism then corresponds to what Myerson (1982) terms a direct mechanism.

We provide two examples in such a setting. Example 1 shows that an equilibrium outcome in simple mechanisms that is strongly robust (as defined by Peters, 2001) is no longer sustained as an equilibrium when principals can deviate to richer communication schemes. When effort is directly contractible, Han (2007) shows that a strongly robust pure strategy equilibrium of a game in which only simple mechanisms are allowed continues to be a strongly robust equilibrium when principals can offer indirect mechanisms. Our example thus demonstrates that his result does not extend to the case of non-contractible effort. That is, in a multi-principal multi-agent model with moral hazard, payoff profiles supported via an equilibrium in simple mechanisms may not survive the introduction of a more complex communication scheme by a principal.

Example 1 also suggests a failure of the revelation principle in our setting: the probability distribution over principals' decisions in an equilibrium in which principals offer indirect communication mechanisms cannot be replicated by restricting principals to the use of simple mechanisms. This failure differs from those identified in multi-principal games with a single

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agent (see Martimort and Stole, 2002, and Peters, 2001), since we explicitly allow principals to make recommendations on agents' effort choices.

However, in our first example, there is an equilibrium in simple mechanisms which replicates the payoffs that both principals and agents obtain in the equilibrium of the indirect mechanism game. We therefore construct a second example which demonstrates a more conclusive failure of the revelation principle: a payoff profile is achievable in equilibrium when one principal can offer an indirect mechanism, but not when all principals are restricted to simple mechanisms.

2. Model

There are 2 principals (denoted as P1 and P2) and 2 agents. There is complete information about agents' types. Each principal j chooses an allocation $y_j \in Y_j$. Each agent i chooses an unobservable effort $e^i \in E^i$. The profile of allocations and efforts, (y_1, y_2, e^1, e^2) results in final payoffs to each principal and agent.

We use the communication structure for principal-agent models introduced by Myerson (1982). At stage 1, principals offer mechanisms. A mechanism offered by principal j is defined as $\gamma_j = (M_j, R_j, \pi_j)$. Here, $M_j = M_j^1 \times M_j^2$ is a message space, from which agents can choose messages to send to principal j, and $R_j = R_j^1 \times R_j^2$ represents a recommendation space, from which private recommendations to agents will be made. The mechanism of principal j also includes a stochastic choice rule $\pi_j : M_j \to \Delta(Y_j \times R_j)$. At stage 2, each agent i sends a message $m_i^j \in M_i^j$ to each principal j. At stage 3, each principal j chooses one element from the distribution $\pi_j(m_j^1, m_j^2)$, and privately communicates the recommendation r_j^i to each agent i. Each agent i chooses her effort e^i at stage 4. Finally, at stage 5, payoffs are realized.

Note that principal j is allowed to choose a lottery over allocations and recommendations for any message array m_j. However, it is standard to consider the mechanism γ_j as a pure strategy for principal j, with a mixed strategy consisting of a randomization over such mechanisms (see, for example, Peters, 2001).

Standard models of moral hazard allow a principal to observe an outcome that is correlated with agents' effort, and to condition compensations (or allocations in our language) on the outcome. Our model is easily generalized to this framework. In our examples, the outcome space may be thought of as a singleton, which allows us to directly consider allocations rather than mappings from outcomes to allocations.

In our framework, a simple mechanism for principal j is defined as follows. For each agent, the principal sets a singleton message space and directly suggests the action the agent should take.[1] That is, $|M_j^i| = 1$ and $R_j^i = E^i$ for every i. Thus, a simple mechanism corresponds to a "direct mechanism" as defined by Myerson (1982).[2] We refer to any mechanism in which, for any agent i, either $|M_j^i| > 1$ or $R_j^i \neq E^i$, or both, as an indirect mechanism.

The set of feasible message spaces for principal j is denoted as $\mathcal{M}_j$, with $\mathcal{R}_j$ the set of feasible recommendation spaces. In a simple mechanism game, principals are restricted to using simple mechanisms, so for each principal j and agent i, $|M_j^i| = 1$ for every $(M_j^1, M_j^2) \in \mathcal{M}_j$, and $\mathcal{R}_j = E^1 \times E^2$. In an indirect mechanism game, these sets are rich enough to allow for simple or indirect mechanisms to be chosen.

In this context, one way to state the revelation principle is as follows. Fix the sets $\mathcal{M}_j$ and $\mathcal{R}_j$ so as to allow each principal j the choice of an indirect mechanism. Consider any equilibrium of the resultant game. Then, there is an equilibrium of the simple mechanism game that reproduces the same probability distribution over allocations and efforts.[3] Note that in a multi-principal environment with moral hazard, the notion of incentive compatibility is not well-defined, since agents may receive conflicting recommendations from the two principals. Thus, rather than impose incentive compatibility, we only require that agents play a continuation equilibrium of the game once the mechanisms are chosen.

A weaker statement of the revelation principle is as follows. Again, fix the sets $\mathcal{M}_j$ and $\mathcal{R}_j$ so as to allow each principal j the choice of an indirect mechanism and consider any equilibrium of the resultant game. Then, there is an equilibrium of the simple mechanism game which delivers the same expected payoffs to each principal and agent. However, the distribution over allocations and efforts may be quite different in the two equilibria.

Following Peters (2001), we define strong robustness as follows. In the competing mechanism game, fix spaces $\mathcal{M}_j$ and $\mathcal{R}_j$ for each principal j. Since the allocation space Y_j is also fixed, this in turn fixes the set of feasible choice rules and hence of feasible mechanisms, Γ_j. Suppose that for any mechanisms $(\gamma_1, \gamma_2) \in \Gamma_1 \times \Gamma_2$ chosen by principals, agents play some continuation equilibrium (in which they choose messages at stage 2 and efforts at stage 4). An equilibrium of the simple (indirect) mechanism game is strongly robust if no principal j can improve his own payoff by offering any simple (indirect) mechanism $\gamma_{\tilde{j}} \in \Gamma_j$, even if agents coordinate on the continuation equilibrium that maximizes the payoff of principal j.

[1] Setting a singleton message space is equivalent to not asking for a message, and allows us to preserve the interpretation of π as a mapping that depends on messages.

[2] A different route to define direct mechanisms is suggested in Epstein and Peters (1999), who include the communication about other principals' mechanisms in the set of messages available to each single agent.

[3] This way of defining the revelation principle may be seen as a natural extension of the notions suggested in the literature on competing mechanisms in the presence of a single agent; see Martimort and Stole (2002) and Peters (2001).