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Perfect competition in an oligopoly (including bilateral monopoly) ☆

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In honor of Martin Shubik

Abstract

We show that if limit orders are required to vary smoothly, then strategic (Nash) equilibria of the double auction mechanism yield competitive (Walras) allocations. It is not necessary to have competitors on any side of any market: smooth trading is a substitute for price wars. In particular, Nash equilibria are Walrasian even in a bilateral monopoly.

JEL classification: C72; D41; D44; D61

Keywords: Limit orders; Double auction; Nash equilibria; Walras equilibria; Mechanism design

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1. Introduction

As is well known Walrasian analysis is built upon the Hypothesis of Perfect Competition, which can be taken as in Mas-Colell (1980) to state: "...that prices are publicly quoted and are viewed by the economic agents as exogenously given." Attempts to go beyond Walrasian analysis have in particular involved giving "a theoretical explanation of the Hypothesis itself" (Mas-Colell, 1980). Among these the most remarkable are without doubt the 19th century contributions of Bertrand, Cournot and Edgeworth (for an overview, see Stigler, 1965). The Cournot approach was explored intensively, in a general equilibrium framework, in the symposium issue entitled "Non-cooperative approaches to the theory of perfect competition" (Journal of Economic Theory, vol. 22 (1980)).

The features common to most of the symposium articles are:

(a) The strategies employed by the agents are of the Cournot type, i.e., consist in quoting quantities.
(b) The (insignificant) size of any agent relative to the market is the key explanatory variable for the tendency of strategic behavior to approximate perfect competition and, in its wake, to lead to Walrasian outcomes (Mas-Colell, 1980, p. 122).

The extension of pure quantity strategies from Cournot's partial equilibrium model of oligopoly to a general equilibrium framework, however, does raise questions. Underlying the Cournot model is a demand curve for the particular market under consideration which enables the suppliers to relate quantities, via prices, to expected receipts. If such a close relationship is not provided by the market, then it seems more natural to us that an agent will no longer confine himself to quoting quantities, i.e., to pure buy-or-sell market orders. To protect himself against "market uncertainty—or illiquidity, or manipulation by other agents" (Mertens, 2003), he will also quote prices limiting the execution of those orders, consenting to sell q units of commodity j only if its price is p or more, or buy $\tilde{q}$ units only if its price is $\tilde{p}$ or less. By sending multiple orders of this kind an agent can approximate any monotone demand or supply curve in a market by a step function, as was done in Dubey (1982, 1994). Here we go further and give each agent *full* manoeuvrability. He places a continuum of infinitesimal limit-price orders, which in effect enables him to send any monotone, continuous demand or supply curve for each commodity.[1] The upshot is a striking result: provided only that all commodity markets are "active" (i.e., there is positive trade in them), and no matter how thin they are, *strategic* (*Nash*) *equilibria* (*SE*) *coincide—in outcome space—with competitive* (*Walras*) *equilibria* (*CE*). Our result thus provides a rationale, based on strategic competition, for Walrasian outcomes even in the case of a bilateral monopoly. This brings it in sharp contrast to Dubey (1982, 1994), where it was necessary to allow for price wars via competition on both sides of each market (in the sense of there being at least two active buyers and two active sellers for each commodity) in order to conclude that SE are CE.[2] The key point of our paper is that *continuous trading is a substitute for price wars and yields perfect*

[1] It must be emphasized that our model is based on *decentralized* markets, and is therefore an order-of-magnitude simpler than that of Mertens (2003), where cross-market limit orders are permitted. SE form a large superset of CE in Mertens' model—for instance, the SE of Shapley's windows model (see Sahi and Yao, 1989) are also SE there.

[2] Indeed, in Dubey's model, the coincidence of SE and CE fails drastically if there is a monopolistic agent in any market. In particular, in a bilateral monopoly, every individually rational and strategically feasible allocation is sustained by SE!