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Congestion games with load-dependent failures: Identical resources ☆

Michal Penn [a], Maria Polukarov [b,*,1], Moshe Tennenholtz [c,a]

[a] Faculty of Industrial Engineering and Management, Technion, Haifa 32000, Israel
[b] School of Electronics and Computer Science, University of Southampton, SO17 1BJ, Southampton, UK
[c] Microsoft Israel R&D Center, 13 Shenkar Street, Herzeliya 46725, Israel

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ABSTRACT

We define a new class of games, *congestion games with load-dependent failures* (CGLFs). In a CGLF each player can choose a subset of a set of available resources in order to try and perform his task. We assume that the resources are identical but that players' benefits from successful completion of their tasks may differ. Each resource is associated with a cost of use and a failure probability which are load-dependent. Although CGLFs in general do not have a pure strategy Nash equilibrium, we prove the existence of a pure strategy Nash equilibrium in every CGLF with nondecreasing cost functions. Moreover, we present a polynomial time algorithm for computing such an equilibrium.

1. Introduction

Congestion games have become a major issue of study for the interplay between game theory and computer science, and are widely discussed in the multi-agent systems and the electronic commerce literature. In a classical congestion game, as defined by Rosenthal (1973), there exists a set of n players and a set of m resources. A strategy for a player is associated with a subset of the resources. In general, each player has his own set of possible strategies. Notice that a strategy is associated with a subset of the resources and not with a particular resource. Each resource is associated with a resource utility function, which determines the utility of a player who selected this resource as a function of the number of players using it. Given a strategy profile, a single strategy for each player, it determines the number of players who will be using each resource. The payoff for a player will be the sum of his utilities from the resources he has selected. In many cases the term "resource utility function" is replaced by the term "resource cost function", to reflect the nature of the particular application; the definition however remains as the one discussed above. In many applications discussed in the literature the resource utility function is decreasing as a function of the number of users (or, the resource cost function is increasing). This may reflect situations where a resource is a service provider whose costs per user are increasing due to competition on *internal* resources. In other applications, such as cost sharing, the resource cost functions are decreasing, reflecting cooperation among the users.

Consider the application of congestion games to a service industry. Let us assume that each resource is a law firm. A client pays the firm on an hourly basis. The law firm employs many lawyers that have access to a small number of shared

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* Corresponding author.
E-mail addresses: mpenn@ie.technion.ac.il (M. Penn), mp3@ecs.soton.ac.uk (M. Polukarov), moshet@microsoft.com, moshet@ie.technion.ac.il (M. Tennenholtz).

[1] The work was done when the author was a PhD student at the Technion, Haifa 32000, Israel.

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resources (starting from local printers, and ending up with sources of information). When a client arrives he is assigned to a particular lawyer, where each lawyer can handle only a fixed small capacity of clients. Hence, when clients arrive the lawyers find themselves competing on a set of available resources, the time spent per client increases as a function of the number of clients of the firm, and the cost per client is monotonically increasing in the number of clients. Needles to say, this situation is typical to many canonical examples in the service industry, such as law firms, accounting firms, and private detective agencies.

Consider now congestion games as discussed above with the following addition. Assume that each resource may fail with probability f, where this probability may depend on the number of players who have selected it: the more players have selected this resource the higher the probability it will fail to deliver. Player i which has a set of available strategies Σ_i (recall that each strategy is a subset of the resources), is paid an additional utility v_i if at least one of the resources in his selected strategy, $\sigma_i \in \Sigma_i$, does not fail. Obviously, this defines a strict extension of congestion games; classical congestion games are obtained when selecting $f = 1$ or $v_i = 0$ for every player. In this paper we consider a restriction on this strict generalization of congestion games: all resources are taken to be identical, and all possible subsets of resources are available as possible strategies for each player. We call this model "Congestion Games with Load Dependent Failures" [CGLFs].[2]

Let us now return to the motivation behind CGLFs. Recall the application of congestion games to the service industries, and consider for example the situation in the private detective agencies sector. In this case, a typical client may approach several private detective agencies, and will be charged by all; moreover, typically, the client is interested in the verification of a particular question, and will benefit as long as at least one of the agencies is able to deliver the answer; needless to say, delivery by an agency may fail, and may depend on the number of clients served by the agency. Such situations fit squarely into the CGLF setting. Of course, one may wish to re-visit the model, and consider various possible modifications; CGLFs however are the first attempt to handle such fundamental issues. The reader should be careful about the use of the term "resource"; in the service industry examples each resource is a firm in the formal model; the fact that resource cost functions may be increasing is a result of the competition on internal resources within the firm, which are not part of the model.

In a CGLF, the resource cost function may be increasing as in the service industry example, or decreasing. The latter typically reflects situations when there is price reduction due to economy of scale (for example, when the resources in the game represent buyer clubs which enable users to share their costs). An important modeling issue is whether costs are incurred by resources which fail to deliver; that is, would one need to pay for the service regardless of whether a success has been declared, or only in the case of success. Indeed, both options are reasonable. For simplicity, we will assume one of the options, and later discuss the other one. Both options lead to similar results.

In a previous paper we discussed the model of "Congestion Games with Failures" [CGFs] (Penn et al., 2005). Although the terms CGF and CGLF may sound similar, these models refer to very different classes of situations. In a CGF players care about the delay caused by using a set of alternative resources, and therefore the payoff of a player is determined by the minimum of delays of the set of selected resources (and by incompletion costs). Notice that this model does not define an extension of congestion games since it does not consider the additivity of costs across selected resources. Therefore, CGFs are an interesting model that model very different situations than CGLFs, have very different motivation, and do not refer to an extension of congestion games. Indeed, the proof techniques discussed for CGFs and CGLFs are completely different. In fact, if we take the probability of failure to be a constant, as is done in CGFs, then CGLFs will possess a potential function and therefore can be viewed as congestion games (Monderer and Shapley, 1996). While this is a technical observation, the major differences stem from the fundamentally different assumptions discussed above. It is also worth noticing that CGLFs should not be viewed as an extension of simple network games, where the network is a set of parallel identical links. They indeed refer to a strict extension of congestion games, although our study deals only with the identical-resources complete strategy spaces version.

In this paper we introduce CGLFs and show that:

- CGLFs and, in particular, CGLFs with nondecreasing cost functions, do not always admit a potential function. Therefore, they are not isomorphic to congestion games. Nevertheless, if the failure probabilities are constant (do not depend on the congestion) then a potential function is guaranteed to exist.
- CGLFs and, in particular, CGLFs with decreasing cost functions, do not always possess pure strategy Nash equilibria. However, as we show in our main result, there exists a pure strategy Nash equilibrium in any CGLF with nondecreasing cost functions. Moreover, we present an efficient algorithm for constructing such an equilibrium in any given CGLF with nondecreasing costs.[3] The time complexity of our algorithm is $O(n^2m + nm^2)$, where n and m represent the number of players and resources, respectively.

While the work on CGFs is the most related to our current study, our work can be viewed as part of the literature extending upon congestion games. In particular, Leyton-Brown and Tennenholtz (2003) extended the class of (simple) congestion

[2] Although one may wish to denote general congestion games with load-dependent failure by CGLFs, and use a different notation for the special case of identical resources, for simplicity of exposition we have chosen to use CGLF for the special case of identical resources studied in this paper.

[3] The preliminary version of the paper (Penn et al., 2007) does not include this part.