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Competitive environments and protective behavior

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ABSTRACT

The class of two-person competition games is introduced and analyzed. For any game in this class the set of Nash equilibria is convex and all Nash equilibria lead to the same payoff vector. Competition games are compared to other competitive environments such as unilaterally competitive games and rivalry games. Moreover, protective behavior within competitive environments is analyzed. For matrix games it is known that protective strategies profiles exactly correspond to proper equilibria. It is shown that this result can be extended to the class of unilaterally competitive games.

1. Introduction

In a matrix game, the set of Nash equilibria exhibits the following well-known characteristics: all Nash equilibria lead to the same payoff vector, the set of Nash equilibria is a convex set, and, as a consequence, equilibrium strategies are exchangeable. Moreover, the set of proper equilibria (Myerson, 1978) can be alternatively characterized by the Dresher procedure (Dresher, 1961) or by the notion of protectiveness (Fiestras-Janeiro et al., 1998). These characterizations do not hold in the more general class of bimatrix games.

In the literature several classes of bimatrix games have been considered in which the Nash equilibrium set retains the first three characteristics. We mention the class of almost strictly competitive games (Aumann, 1961), the class of strictly competitive games (Friedman, 1983), the class of unilaterally competitive games (Kats and Thisse, 1992), and perhaps less known the class of rivalry games (Rauhut et al., 1979). Roughly speaking, the above classes of bimatrix games (A, B) have in common that they require specific relationships between the best reply structure of (A, B) and $(-B, -A)$. In the same spirit this paper introduces the class of competition games. A bimatrix game (A, B) is a competition game if the best reply structures of (A, B) and $(-B, -A)$ are such that the Nash equilibria of both games coincide.

Any strictly competitive game is unilaterally competitive; any unilaterally competitive game is a rivalry game; and, any rivalry game is almost strictly competitive. For every game in any of these classes, all Nash equilibria have identical payoff vectors and the set of Nash equilibria is a convex set. This paper shows that the class of competition games is in between the class of rivalry games and the class of almost strictly competitive games.

We focus on the possible relation between proper equilibria and protective strategy profiles in competitive environments à la Fiestras-Janeiro et al. (1998). It turns out that the set of protective strategy profiles coincides with the set of proper

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equilibria in the class of unilaterally competitive games. In the class of rivalry games, protective strategy profiles are (perfect) equilibria but not necessarily proper equilibria. In the process we analyze relations between the equilibria of bimatrix games (A, B) and the equilibria in the related matrix games $(A, -A)$ and $(B^t, -B^t)$ for different classes of competitive environments. The specific distinction of the class of competition games helps to clarify the differences in the relations. These relations will be used in the proofs of the main theorems.

The paper is organized as follows. Basic definitions are provided in Section 2. In Section 3 we define the different competitive environments under consideration and describe the relationships between them. Section 4 is devoted to the relation between protective strategy profiles and proper Nash equilibria.

2. Preliminaries

A *bimatrix game* (A, B) is a two person game $(\Delta(S_1), \Delta(S_2), \pi_1, \pi_2)$ in strategic form, where A and B are two $m \times n$ matrices, $S_1 = \{e_1, \ldots, e_m\}$ and $S_2 = \{f_1, \ldots, f_n\}$ are the pure strategy sets of player 1 and player 2, respectively, and the payoff functions π_1 and π_2 are defined as[1]

$$\pi_1(p, q) = pAq \quad \text{and} \quad \pi_2(p, q) = pBq$$

for every pair of mixed strategies $p \in \Delta(S_1)$ and $q \in \Delta(S_2)$. A bimatrix game (A, B) where $B = -A$ is called a *matrix game* and it is usually denoted by A.

Let us consider a bimatrix game (A, B). A combination $(p, q) \in \Delta(S_1) \times \Delta(S_2)$ is called a *strategy profile*. For any $p \in \Delta(S_1)$, the set

$$B_2(p) = \Big\{\bar{q} \in \Delta(S_2) \;\Big|\; pB\bar{q} = \max_{q \in \Delta(S_2)} pBq\Big\}$$

is the *set of best replies* of player 2 against the strategy p of player 1; the set

$$A_2(p) = \Big\{\bar{q} \in \Delta(S_2) \;\Big|\; pA\bar{q} = \min_{q \in \Delta(S_2)} pAq\Big\}$$

gives us the *set of antagonistic replies* of player 2 with respect to the strategy p of player 1. With the obvious modifications one defines the sets $B_1(q)$ and $A_1(q)$ for any $q \in \Delta(S_2)$.

We say that $\bar{p} \in \Delta(S_1)$ is a *completely mixed strategy* if $\bar{p}_i (= \bar{p}(e_i)) > 0$ for all $i = 1, \ldots, m$. Analogously, we define completely mixed strategies for player 2.

A strategy profile $(\bar{p}, \bar{q})$ is called a *Nash equilibrium* if

$$\bar{p}A\bar{q} \geqslant pA\bar{q} \quad \text{for all } p \in \Delta(S_1) \quad \text{and} \quad \bar{p}B\bar{q} \geqslant \bar{p}Bq \quad \text{for all } q \in \Delta(S_2).$$

Hence, $(\bar{p}, \bar{q})$ is a Nash equilibrium for (A, B) if and only if $\bar{p} \in B_1(\bar{q})$ and $\bar{q} \in B_2(\bar{p})$.

A strategy profile $(\bar{p}, \bar{q})$ is called a *twisted equilibrium* (Aumann, 1961) if

$$\bar{p}A\bar{q} \leqslant \bar{p}Aq \quad \text{for all } q \in \Delta(S_2) \quad \text{and} \quad \bar{p}B\bar{q} \leqslant pB\bar{q} \quad \text{for all } p \in \Delta(S_1).$$

Hence, $(\bar{p}, \bar{q})$ is a twisted equilibrium for (A, B) if and only if $\bar{p} \in A_1(\bar{q})$ and $\bar{q} \in A_2(\bar{p})$. $\mathrm{E}(A, B)$ will denote the set of Nash equilibria and $\mathrm{TE}(A, B)$ the set of twisted equilibria of (A, B). Notice that the twisted equilibria of (A, B) exactly correspond to the Nash equilibria of the bimatrix game $(-B, -A)$.

The following example shows that $\mathrm{E}(A, B) \cap \mathrm{TE}(A, B)$ can be empty.

Example 2.1. Consider the bimatrix game (A, B) defined by

$$A = \begin{pmatrix} 3 & 5 \\ 3 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}.$$

Then, $\mathrm{E}(A, B) = \{(e_1, f_2)\}$ but (e_1, f_2) is not a twisted equilibrium since $A_1(f_2) = \{e_2\}$. Consequently, $\mathrm{E}(A, B) \cap \mathrm{TE}(A, B) = \emptyset$.

Next, we recall the definitions of the main concepts that we will use later on. Let (A, B) be a bimatrix game.

A strategy profile (p, q) is a *proper equilibrium* (Myerson, 1978) if there exist sequences $\{(p^k, q^k)\}_{k \in \mathbb{N}}$ of strategy profiles and $\{\varepsilon^k\}_{k \in \mathbb{N}}$ of real numbers such that $\lim_{k \to \infty} \varepsilon^k = 0$, $\lim_{k \to \infty} (p^k, q^k) = (p, q)$, and for all $k \in \mathbb{N}$,

(i) $\varepsilon^k > 0$ and (p^k, q^k) is a completely mixed strategy profile,
(ii) for all $e_i, e_j \in S_1$ such that $e_iAq^k < e_jAq^k$ we have $p_i^k \leqslant \varepsilon^k p_j^k$, and
(iii) for all $f_r, f_s \in S_2$ such that $p^kBf_r < p^kBf_s$ we have $q_r^k \leqslant \varepsilon^k q_s^k$.

[1] We write pAq instead of p^tAq and pBq instead of p^tBq.