Games and Economic Behavior 67 (2009) 292–299

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Private provision of discrete public goods ☆

Miltiadis Makris *

University of Leicester, United Kingdom

ARTICLE INFO

Article history:
Received 2 July 2007
Available online 27 November 2008

JEL classification:
C72
D70
H41
D89

Keywords:
Discrete public goods
Warm-glow altruism
Poisson games

ABSTRACT

We investigate private provision of discrete public goods under refund and cost-sharing. If it is commonly believed that individuals may be warm-glow altruists and the group-size is a Poisson random variable, then the equilibrium distribution of collected contributions is uniquely determined. If composition uncertainty is very small and the expected group-size sufficiently large, the distribution of contributions can be described by concentrating in a symmetric mixed-strategy equilibrium. As the expected group-size increases, the probability a randomly selected player contributes and the associated expected number of contributions converge to the corresponding ones in the symmetric mixed-strategy equilibrium of the game with no uncertainty.

1. Introduction

Private provision of public goods is often claimed to be inefficient—especially, in a large economy.[1] However, discrete public goods may be efficiently provided. Discrete public goods are goods that are produced if and only if a certain level of contributions is reached. Examples of such problems are: putting up a streetlight, collecting money for an office coffee club, setting up a lobby given the political environment, signing a petition, etc. In such cases, it is well known that, when the number of potential contributors and their preferences are common knowledge, multiplicity of Nash equilibria may emerge regardless of the group-size (see, for instance, Palfrey and Rosenthal, 1984). Typically, there are two inefficient symmetric equilibria: one is a mixed-strategy and the other is with zero contributions. There is also an efficient equilibrium outcome in asymmetric pure-strategies, where just enough contributions are made so that the public good is certainly provided.

In this paper, however, we show that if it is common knowledge that (a) some players, the number of whom is uncertain, have contributing as a dominant strategy, *and* (b) the total number of players is drawn from a Poisson distribution, then the equilibrium distribution of the number of contributions is *uniquely* specified *regardless* of the expected group-size. The probability a randomly selected player contributes is also uniquely determined, and is *decreasing* with the expected number of potential contributors. Furthermore, if the expected group-size is sufficiently large and the likelihood of having a dominant strategy to contribute becomes very small, then the unique probability distribution of collected contributions

☆ I would like to thank an Advisory Editor and two anonymous referees for very helpful and constructive comments that have improved the paper significantly. I am also indebted to Dieter Balkenborg for the very helpful and insightful long discussions. Many thanks should also go to Clare Leaver for the very helpful discussions at the early stages of this work. I would also like to thank Francesco Giovannoni, Joel Watson, Patrick Francois, Amrita Dhillon and participants in seminars at Exeter, UCL, Verona, and in ESRC Conferences in Game Theory and Public Economics for their suggestions and helpful comments on earlier versions of this paper. The usual disclaimer applies.

* Address for correspondence: Department of Economics, University of Leicester, University Road, Leicester LE17RH, UK. Fax: +44 (0)116 252 5351.
E-mail address: M.Makris@le.ac.uk.

[1] See, for instance, Gradstein (1998).

0899-8256/$ – see front matter
doi:10.1016/j.geb.2008.11.003

can, without loss of generality, be described by concentrating in an equilibrium where all players without a dominant strategy to contribute choose the *same* mixed-strategy. Finally, we show that if the likelihood of having a dominant strategy to contribute becomes very small, then as the expected group-size increases, the probability a randomly sampled player contributes and the associated expected number of contributions converge to the corresponding ones in the symmetric mixed-strategy equilibrium of the game with no uncertainty.

Our work provides an instance in which the predictive power of a model increases after introducing sufficient uncertainty about fundamentals. However, in our case it is *essential* that introduced uncertainty is about *both* the preferences and the number of players.

Our work provides also a *theoretical justification* for a commonly adopted appeal to symmetry in arguing in favour of restricting attention to the symmetric equilibria of the contribution game with no uncertainty. Dixit and Olson (2000), for instance, focus on these equilibria due to "the difficulty of the coordination that is required ... when a subset of a larger group is designated to do one thing and the rest another." This appeal to symmetry has often been criticised in that the "difficulty of coordination" reflects the symmetry of players,[2] and does not imply a symmetric equilibrium. Our results, however, suggest that restricting attention to the symmetric mixed-strategy equilibrium of the game with no uncertainty could be thought of as a simplifying shortcut of discussing instead a model like the one here, where there is very small uncertainty about the players' preferences and the expected group-size is sufficiently large.

Postulating population uncertainty, follows the suggestion by Myerson (1998, 2000) that in large games it may be more realistic to admit some uncertainty about the number of other players in the game. For instance, in large societies, it may be prohibitively expensive to collect the necessary information for who all the stakeholders are. Given, then, the convenient properties associated with the Poisson distribution (see Myerson, 1998), we model the voluntary-provision problem as a Poisson game. As a complementary justification, suppose that identities of stakeholders are indeed common knowledge but also that contributions must be collected by a given time. However, potential stakeholders may be ill, postmen may be on strike or computer networks may be down. As a result, in a large environment, stakeholders should actually view the number of players *in* the game as a Poisson random variable.

In our model, some individuals may have a dominant strategy to contribute due to them deriving sufficient satisfaction from the action of contributing itself. That is, we postulate that some stakeholders may be 'warm-glow altruists.'[3]

Our work is related to a number of papers that focus on equilibrium selection either via myopic strategy-revision processes that are subject to some noise[4] or via the 'global games' approach, where it is assumed that players receive privately observed noisy signals about the actual game they are playing.[5] Interestingly, this strand of research emphasises that, depending on the fundamentals of the problem, the selected play is characterised by a probability of provision that equals either one or zero. In contrast, our model predicts, without having to specify a detailed process of equilibrium selection, that the public good will be provided with a probability that, in certain cases, lies *strictly* between zero and one. Our work is also related to Makris (2008), where population uncertainty is also deployed by means of the Poisson distribution. The focus there, however, is on games with strategic complementarities. Moreover, there is no uncertainty over the composition of the group. The result there is that the equilibrium of such games *is* unique, but, in contrast to the prediction here, only when the expected number of players is sufficiently *small*.

The organisation of the paper is as follows. Next section describes the model. Section 3 discusses equilibria under the assumption of population certainty. Section 4 investigates equilibria under the assumption of population uncertainty, with and without composition uncertainty. Finally, Section 5 discusses our results and concludes.

2. The model

We consider an economy with N citizens. Each citizen has an endowment of y units of the private good and derives utility from the consumption of a discrete public good $g \in \{0, 1\}$. The cost of provision, i.e. of $g = 1$, is k. Citizens must decide whether to participate in the production of the public good or not. The income net of any contributions is denoted with ω. We denote the choice of each citizen by $c \in \{0, 1\} \equiv \mathcal{C}$, where $c = 1$ denotes the decision to participate/contribute. Moreover, x_c denotes the number of citizens who have chosen action c.

In this paper we consider voluntary provision of the public good under the institutional arrangement of cost sharing between participants/contributors and that unused contributions are returned.[6] We refer to this as the full refund set-up. We focus on this particular class of games for a number of reasons. First, Dixit and Olson (2000) is a special case of this class, and we are ultimately interested to investigate whether their 'pragmatic' argument, often used in other environments as well, can find theoretical support. Second, we are also interested in the question of whether government intervention

[2] See, for instance, Myatt and Wallace (2004).

[3] See Andreoni (1988). For similar modeling choices, see also Palfrey and Rosenthal (1988) and Hindriks and Pancs (2002).

[4] See Young (1993), Kandori et al. (1993) and Myatt and Wallace (2004).

[5] See Myatt and Wallace (2002), for a game with two players when contributions are not returned.

[6] The refund rule presumes that contributions are not physical in nature. The refund rule also presumes the presence of a collector who is willing to reimburse, whenever the public good is not produced, any monetary sunk costs incurred by contributors. Such a collector could, for instance, be an entrepreneur who receives a (sufficiently high) fixed payment whenever the public good is provided in exchange for her organising the collection of contributions. So, in general, production costs may also include the payment to the collector.