



Budget-constrained sequential auctions with incomplete information [☆]

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ABSTRACT

I study a budget-constrained, private-valuation, sealed-bid sequential auction with two incompletely-informed, risk-neutral bidders in which the valuations and income may be non-monotonic functions of a bidder's type. Multiple equilibrium symmetric bidding functions may exist that differ in allocation, efficiency and revenue. The sequence of sale affects the competition for a good and therefore also affects revenue and the prices of each good in a systematic way that depends on the relationship among the valuations and incomes of bidders. The sequence of sale may affect prices and revenue even when the number of bidders is large relative to the number of goods. If a particular good, say α , is allocated to a strong bidder independent of the sequence of sale, then auction revenue and the price of good α are higher when good α is sold first.

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1. Introduction

Much of the existing theoretical work on auctions concentrates on the allocation of a single good.¹ However, in actual auctions, several heterogeneous goods are often allocated sequentially. If there is no link among the goods then one may be able to apply the single-good analysis repeatedly. However, such a link may arise if budget constraints limit a bidder's ability to bid for later goods when earlier prices deplete the bidder's limited resources.

Individual bidders whose valuations derive from consumption (rather than resale) may be budget-constrained. But the relevance of budget constraints extends well beyond this case. A theoretical literature argues generally that the existence of agency problems implies that firms are effectively budget-constrained in their investment decisions.² An empirical literature supports this idea.³ In the context of auctions, even firms that are buying to re-sell may effectively be budget-constrained if the cost of borrowing increases with the amount borrowed⁴ (a standard assumption in the finance literature) or if capital market imperfections result in budgets for projects being determined on a yearly basis, so that the firms allocate only a

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¹ For a survey of the literature, see Klemperer (1999).

² See Lewis and Sappington (1989a, 1989b), Hart and Moore (1995) and Clementi and Hopenhayn (2006).

³ See Fazzari and Athey (1987), Fazzari et al. (1988), Whited (1992), Fazzari and Petersen (1993), Love (2003) and Clementi and Hopenhayn (2006) for empirical corroboration of budget constraints.

⁴ See Cramton (1995) for a discussion of the budget constraints faced by firms making large investments in the nationwide narrowband PCS auction held in the United States in July 1994.

fixed amount of capital⁵ for the completion of a project. Engelbrecht-Wiggans (1987) shows that budget constraints arise if a bidder is an agent of a principal.

When investments are relatively large then capital market imperfections can lessen the ability of even a large firm to borrow funds. The historic auction of radio spectrum by the FCC in the USA is a good example of an auction in which the investments are relatively large. Cramton (1995) finds it realistic to assume that all firms in PCS (personal communicating services) auctions face budget constraints.⁶ As he explains, bidders must raise funds before the auction starts when they do not know exactly how much they will need. Given that fund-raising is time-consuming and costly, he argues that it is reasonable to assume that firms that come to such auctions are budget-constrained. In addition, only forty per cent of the narrow band PCS spectrum was for sale in the first spectrum auction held by the FCC, so that, though each spectrum auction was simultaneous, goods were allocated sequentially across auctions as well as simultaneously within an auction.

I find that the order of sale affects revenue in a private value budget-constrained sequential auction with imperfect information in which bids are continuous. The order of sale affects revenue and prices whether information is perfect and bids are discrete or whether information is imperfect and bids are continuous. The intuition derives from the fact that once good 1 is sold, there is an option to win good 2. The value of the option depends on demand for good 2 which in turn depends on the order of sale. Benoît and Krishna (2001) show that in a complete information common value auction of two goods and three budget-constrained bidders, selling the more highly valued good first always generates the highest revenue. Their result extends to two goods and n budget-constrained bidders since only the top three incomes are relevant. However, it is easy to generate budget-constrained sequential common value auctions in which selling the most highly valued of three goods does not generate the highest revenue.⁷

The preceding paragraph illustrates that the relationship between the order of sale and revenue is unclear. The question that I address in this paper is whether any systematic rules govern the relationship among the prices of a good, the revenue, and the order of sale when the valuations are similar and when the income exceeds the valuation of each good. In Section 7, I provide a restricted set of auctions in which selling the more highly valued good first raises more revenue than selling it second.

I study a budget-constrained version of the benchmark model of a private-valuation sealed-bid sequential auction in which two risk-neutral bidders bid for two goods and information is incomplete.⁸ When information is complete, revenue is affected, in a systematic way,⁹ by the price-formation rule (i.e. the rule that specifies the price as a function of the bids). Assuming information is incomplete does not change this. In order to isolate the pure effect of the budget constraints on the prices of goods relative to their order of sale and the price-formation rule, I restrict to a world in which the expected revenue is constant within a class of price-formation rules that includes 1st and 2nd price rules (as would happen if there were only a single good and no effective budget constraints). In this world, I find that the auction revenue depends on the sequence of sale, that the price of a good depends on its position in the sequence of sale, and that this dependence has a natural interpretation.

In a budget-constrained sequential auction of two goods in which a bidder's type determines the value of the bidder's valuation and income functions, I restrict attention to symmetric bidding functions but do not assume monotonicity in a bidder's type. I find that multiple symmetric equilibrium bidding functions may exist that differ with respect to efficiency, revenue and allocation. Whether revenue is maximized or the allocation is efficient depends on the relationship between the bidding function and the valuation and income functions and not on the price rule.

Say that two real-valued functions f and g are *ordinally equivalent* on a common domain S if $f(x) > f(y)$ if and only if $g(x) > g(y)$ for any $x, y \in S$ (that is, they produce a common ranking of the domain elements). Ordinal equivalence can be useful in determining whether an equilibrium bidding function generates the highest revenue or an efficient outcome. An upper bound on the revenue generated is that generated by any bidding function that is ordinally equivalent to the income function (Theorem 2 and Corollary 3). An efficient allocation is generated if the bidding function is ordinally equivalent to the difference in valuation functions (Theorem 2 and Corollary 3). In particular, if the valuation and income functions are increasing in a bidder's type with one valuation function increasing more rapidly than the other, then, under first or second price rules, there exists an equilibrium bidding function that generates maximum revenue and an efficient allocation when the good whose valuation increases more rapidly in a bidder's type is sold first. Thus, even when there are only two goods, selling the highest valued good first need not generate the highest revenue. In particular, if one is auctioning the contents of a household, then selling a wall painting by an unknown artist (for whom bidders' tastes are highly variable) before the used ride-on lawn mower (whose value may be high but publicly known) maximizes revenue.

The price-formation rule may affect the price of a good even if it does not affect revenue (Theorem 7). For example, under a second price rule, a bidder might worry about not being awarded good 2 at a low price and so bid relatively high while, under a first price rule, a bidder might worry about winning good 1 at a high price and so bid relatively low. In

⁵ See Hendricks and Porter (1992) for empirical evidence of capital constraints in land lease auctions.

⁶ As do Burguet and McAfee (2005).

⁷ Benoît and Krishna (2001).

⁸ Benoît and Krishna (2001) and Pitchik and Schotter (1988) consider budget-constrained buyers with complete information. Che and Gale (1998) consider budget-constrained buyers in one-good auctions. Pitchik and Schotter (1986) and Pitchik (1989) consider budget-constrained buyers with incomplete information.

⁹ See Benoît and Krishna (2001), Pitchik and Schotter (1986, 1988).

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