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Consistent representative democracy

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Abstract

We study axioms which define "representative democracy" in an environment in which agents vote over a finite set of alternatives. We focus on a property that states that whether votes are aggregated directly or indirectly makes no difference. We call this property *representative consistency*. *Representative consistency* formalizes the idea that a voting rule should be immune to gerrymandering. We characterize the class of rules satisfying *unanimity*, *anonymity*, and *representative consistency*. We call these rules "partial priority rules." A partial priority rule can be interpreted as a rule in which each agent can "veto" certain alternatives. We investigate the implications of imposing other axioms to the list specified above. We also study the partial priority rules in the context of specific economic models.

JEL classification: D63; D70

Keywords: Social choice; Representative systems; Majority rule; Gerrymandering

1. Introduction

Most democracies use some form of representative democracy. One common type works as follows: voters are grouped into districts, and each district elects its own representative according to majority rule. These representatives then vote, and majority rule is applied to the votes of the representatives. Such a system is referred to as a "single-member district" system. Single-member district representation can result in a different outcome than the majority outcome. The outcome depends on how voters are grouped into districts. It is thus subject to manipulation by those who design the districts–this manipulation is referred to as "gerrymandering." For a prominent recent example, in the 2000 Presidential election of the United States, Gore won the

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popular vote, but due to the way that districts were drawn, Bush ended up winning the electoral vote.

We ask whether or not there *exist* voting rules which can never be gerrymandered. Our primary contribution here is twofold. Firstly, we introduce a condition formalizing the notion that a voting rule should be immune to gerrymandering. To our knowledge, ours is the first work proposing such a notion. Secondly, we investigate the implications of this notion in conjunction with other properties, and characterize all "democratic" voting rules that satisfy it.

To simplify the exposition, we will introduce some notation early on. Let $\mathbb{N}$, the set of natural numbers, index a set of "potential agents." Let $\mathcal{N}$ be the set of finite subsets of $\mathbb{N}$. Let X be a finite set. Elements of X are "alternatives," to be interpreted as policies, candidates, etc. Each agent submits a "vote," or an element of X. A **rule** is a function $f : \bigcup_{N\in\mathcal{N}} X^N \to X$. Each agent in society votes for one of the alternatives. A rule associates a social alternative with every possible group of voters and list of votes that they may submit. As a rule can be applied to any group of voters, it can be applied to districts, or it can be applied to society as a whole.

We specify notions of "direct democracy" and "indirect democracy." In a direct democracy, votes are directly aggregated by a rule. In an indirect democracy, agents are partitioned into districts. Votes are aggregated within each *district*, resulting in a "representative vote" for each district. The winning alternative in each district takes all of the votes for that district. Thus, each agent in society is treated as having voted for his district's representative vote. These representative votes are then aggregated by the rule. We say a rule is representative consistent if these two procedures coincide. To understand the concept, we introduce more notation. For all $N \in \mathcal{N}$, all $y \in X^N$, and all $M \subset N$, let y_M be the restriction of y to X^M. For all $N \in \mathcal{N}$ and all $x \in X$, let x^N be a vector in X^N such that for all $i \in N$, $x_i^N = x$. Formally, representative consistency states that for all $N \in \mathcal{N}$, all partitions $\{N_1, \ldots, N_K\}$ of N, and all $x \in X^N$, $f(x) = f(f(x_{N_1})^{N_1}, \ldots, f(x_{N_K})^{N_K})$.

The notion of indirect vote aggregation in social choice is familiar from the works of Murakami (1966, 1968), Fishburn (1971, 1973), and Fine (1972), although these papers are not concerned with the consistency of indirect vote aggregation and direct vote aggregation. In his famous paper on strategy-proof voting schemes, Moulin (1980) also briefly discusses indirect voting procedures. Other works which are conceptually related (as they deal with a kind of "recursive" application of rules) are Barberà and Jackson (2004), Barberà and Bevia (2002) and Koray (2000).

Representative consistency is a condition of "gerrymandering-proofness."[1] Our primary result confirms the intuition that most voting rules are subject to gerrymandering. However, there are rules which are not. We characterize the class of rules satisfying representative consistency and two other mild requirements (unanimity and anonymity). These rules can be described in a simple way. If the set of alternatives under consideration is X, then any such rule can be associated with a partial order over the set X. The partial order has the characteristic that every pair of alternatives possesses a greatest lower bound (a meet). For any profile of votes, the rule then selects the meet of the alternatives receiving a positive number of votes. We call such a rule a "partial priority rule," as it generalizes a natural notion of prioritizing alternatives.

[1] Thus, we can think of other notions of a rule being immune to gerrymandering. Specifically, by ignoring strategic effects on the part of voters, we are ruling out the possibility that agents may behave differently when placed in different districts. Such behavior may have the effect of "canceling out" attempts at gerrymandering. A weaker notion of *representative consistency* may be desired in this case; such a notion will depend on the assumed equilibrium concept for voter behavior.