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Non-hyperbolic time inconsistency ☆

Han Bleichrodt, Kirsten I.M. Rohde, Peter P. Wakker *

Department of Economics, Erasmus University, Rotterdam, The Netherlands

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Abstract

The commonly used hyperbolic and quasi-hyperbolic discount functions have been developed to accommodate decreasing impatience, which is the prevailing empirical finding in intertemporal choice, in particular for aggregate behavior. However, these discount functions do not have the flexibility to accommodate increasing impatience or strongly decreasing impatience. This lack of flexibility is particularly disconcerting for fitting data at the individual level, where various patterns of increasing impatience and strongly decreasing impatience will occur for a significant fraction of subjects. This paper presents discount functions with constant absolute (CADI) or constant relative (CRDI) decreasing impatience that can accommodate any degree of decreasing or increasing impatience. In particular, they are sufficiently flexible for analyses at the individual level. The CADI and CRDI discount functions are the analogs of the well-known CARA and CRRA utility functions for decision under risk.

JEL classification: D90; C60

Keywords: Hyperbolic discounting; CADI discounting; CRDI discounting; Decreasing impatience; Time preference

1. Introduction

Under stationarity, indifference between a small outcome received soon and a large outcome received later is preserved if both outcomes are equally delayed. Stationarity reflects constant impatience, and is equivalent to time consistency under common assumptions (“stopwatch time,” resetting the zero of time to the moment of decision). Empirical studies have found that stationarity is usually violated (Frederick et al., 2002), with impatience mostly decreasing and not constant. That is, delaying the aforementioned outcomes makes the decision maker less impatient and more willing to wait for the (large and) late outcome. Thus, the indifference turns into a preference for the late outcome, and stationarity is violated.

The most popular discount functions today, the generalized hyperbolic (Loewenstein and Prelec, 1992) and quasi-hyperbolic (Phelps and Pollak, 1968; Laibson, 1997) discount functions, were introduced so as to accommodate

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* Corresponding author at: Econometric Institute, Erasmus University, PO Box 1738, 3000 DR, Rotterdam, The Netherlands. Fax: +31 (0)10 408 91 62.

E-mail address: Wakker@few.eur.nl (P.P. Wakker).

URL: http://people.few.eur.nl/wakker/index.html (P.P. Wakker).

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decreasing impatience. A drawback is that they do not have enough flexibility to accommodate increasing or strongly decreasing impatience, as we will demonstrate. These restrictions make it impossible to fit data at the individual level because there will always be significant fractions of subjects with increasing or strongly decreasing impatience (Abdellaoui et al., 2007; Harrison et al., 2002; Barsky et al., 1997). The impossibility to fit data at the individual level is particularly disconcerting in view of recent advances in neuroeconomics, where typically only a few individuals can be analyzed.

To further clarify the above points, we compare intertemporal choice to choice under risk. Even while risk aversion is the prevailing empirical phenomenon, commonly found for aggregate behavior, at the individual level some risk seekers are always found. Hence, to fit data at the individual level we need utility functions that are flexible enough to accommodate risk seeking. The CARA (constant absolute risk averse) and CRRA (constant relative risk averse) functions can do so, and these functions have, accordingly, been widely used. The logarithmic function[1] $U(x) = b(\ln(a + x))$ cannot accommodate risk seeking and is, therefore, considerably less popular. Without utilities such as CARA and CRRA available, no quantitative analysis of risk attitude at the individual level would be possible. Such a problematic situation exists at present for intertemporal choice. Indeed, some studies that attempted to fit intertemporal preferences at the individual level using (quasi-)hyperbolic discounting typically had to discard some 15 percent of their subjects (Kirby, 1997; Green et al., 1999).

The problem just mentioned is aggravated by some recent empirical studies that even found increasing impatience at the aggregate level (Attema et al., 2008; Gigliotti and Sopher, 2004; Read et al., 2005; Sayman and Öncüler, 2006). Onay and Öncüler (2007) and Chesson and Viscusi (2003) even found concave discounting at the aggregate level, implying strongly increasing impatience. Hence, also at the aggregate level there is an interest in new discount functions that can flexibly accommodate increasing impatience.

The aforementioned findings show that there are problems with hyperbolic and quasi-hyperbolic discounting. This paper introduces two classes of discount functions that avoid the problems mentioned. These classes can accommodate any degree of increasing impatience, and also any degree of decreasing impatience. Hence, they cover all degrees covered by hyperbolic discounting and allow additional degrees on top of those, giving increased flexibility at no cost. Our classes are the intertemporal counterparts of the CARA and CRRA utility functions from decision under risk (or, more precisely, of state-dependent generalizations thereof). They generalize classes of discount functions introduced by Prelec (1989) and Ebert and Prelec (2007). The latter were of a similar mathematical form as our families, but were not rich enough to cover all degrees of increasing or decreasing impatience.

The paper is organized as follows. Basic definitions are in Section 2. Section 3 analyzes the properties of generalized hyperbolic and quasi-hyperbolic discounting. Section 4 presents our classes of discount functions, while their main empirical implications are used to provide preference foundations in Sections 5 and 6. Section 7 derives further properties of our discount functions, demonstrating in particular that they can accommodate any degree of increasing or decreasing impatience. We further generalize the model by allowing for zero and negative utility. Section 8 concludes, and proofs are in Appendix A.

2. Discounting

Throughout this paper we study preferences $\succcurlyeq$ over *timed outcomes* $(t : x) \in T \times X$, where T is a nondegenerate subinterval of $[0, \infty]$ and X is the outcome set. A timed outcome $(t : x)$ is interpreted as the receipt of *outcome* x at *time point* t. The preference notation $\succ$, $\sim$, $\preccurlyeq$, and $\prec$ is as usual.

We assume that preferences $\succcurlyeq$ can be represented by *discounted utility* (*DU*):

$$DU(t : x) = \varphi(t)U(x).$$

That is, $(t : x) \succcurlyeq (s : y)$ if and only if $DU(t : x) \geqslant DU(s : y)$. DU is the *discounted utility*, U is the *utility*, and φ is the *discount function*. Preference conditions for DU were given by Roskies (1965), Krantz et al. (1971, Chapter 7), and Fishburn and Rubinstein (1982) (either only for gains or with φ for losses different than for gains). For brevity, we do not state these conditions but assume DU throughout. Time points are often denoted by s and t, where we usually

[1] We will often use the fact that minus the logarithm of a discount function plays a role in intertemporal choice that in a mathematical sense is similar to utility in decision under risk. Decreasing impatience is the analog of risk aversion and increasing impatience is the analog of risk seeking. Logarithmic utility is the analog of hyperbolic discounting.