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Multi-battle contests ☆

Kai A. Konrad [a,b], Dan Kovenock [c,*]

[a] Social Science Research Center Berlin (WZB), Germany
[b] Free University of Berlin, Germany
[c] Krannert School of Management, Purdue University, West Lafayette, IN 47907, USA

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Abstract

We study equilibrium in a multistage race in which players compete in a sequence of simultaneous move component contests. Players may win a prize for winning each component contest, as well as a prize for winning the overall race. Each component contest is an all-pay auction with complete information. We characterize the unique subgame perfect equilibrium analytically and demonstrate that it exhibits endogenous uncertainty. Even a large lead by one player does not fully discourage the other player, and each feasible state is reached with positive probability in equilibrium (pervasiveness). Expected effort in the component contests may be non-monotonic in the closeness of the race and realized individual effort may exceed the value of the prize by a factor that is proportional to the maximum number of stage victories required.

JEL classification: D72; D74

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1. Introduction

In many contests final success or failure is determined not by the outcome of a single battle, but from multiple battles. For instance, in many contests players compete in a sequence of battles and the player whose accumulated battle victories first reach some predetermined minimum number (which may vary across players), is awarded the prize for victory in the overall contest. We call this structure a *multi-battle contest.*

Multi-battle contests have already received considerable attention in the area of R&D competition. Harris and Vickers (1987) described a patent race as a multi-battle contest: two players expend efforts on R&D in a sequence of single component contests. In each component contest one of the players wins, and the winner is determined as

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* Corresponding author. Fax: +1 765 494 9658.
E-mail address: kovenock@purdue.edu (D. Kovenock).

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a stochastic function of the players' efforts in the respective component contest. The player who is first to win a given number of component contests wins the patent.[1]

In politics, Klumpp and Polborn (2006) have applied multi-battle contests to explain the dynamics of candidate performance and campaign expenditures in the US: presidential primaries. In Klumpp and Polborn's model the winner of each state primary election is determined as a stochastic function of the players' campaign expenditures in that state. The player who is first to win a majority of the state elections wins the party's nomination. They use their findings to examine the parties' incentive to hold the state primary elections sequentially.

Sports is perhaps the most direct application. A tennis match, for instance, consists of a series of single battles. To be victorious, a player needs to win a certain number of sets before his or her competitor does. In the Major League Baseball World Series and the National Basketball Association's Finals playoff series two teams compete in a series of games with the winner being the first team to win four games. Similar examples exist in many other areas of sports competition.[2]

This article examines multi-battle contests in which the component contests are (first-price) all-pay auctions with complete information. An all-pay auction is a simultaneous move game in normal form in which players simultaneously submit bids (or efforts). Each player must pay his own bid (in contrast to winner-pay auctions) and the highest bidder wins the contest or prize (arbitrary tie-breaking rules are permitted). Examination of all-pay auctions dates back to at least Nalebuff and Stiglitz (1983),[3] and the characterization of the one-stage all-pay auction and its applications has extensively been studied in recent years both for complete and incomplete information.[4]

The paper provides a complete characterization of the unique subgame perfect equilibrium of a wide class of symmetric and asymmetric two-player multi-battle contests with all-pay auction components. Players earn potentially different prizes for winning the overall contest and, in addition, identical non-negative intermediate prizes for winning each component contest. Empirically, the counterpart to these intermediate prizes in a patent race might be the benefit obtained from information spillovers or cost reduction in other production processes that arise from winning the component contest. In sports it might be the pride associated with winning individual games or sets. In political primary seasons it might be the prestige or bargaining power arising from controlling the delegates from a given state.

One implication of incorporating intermediate prizes is *pervasiveness*. With positive intermediate prizes, even a large lead by one player does not fully discourage the other player. Both players expend positive effort with positive probability in every component contest, no matter how far behind they might be. As a consequence, each player may win any given contest and each feasible state is reached with positive probability in equilibrium (*pervasiveness*). It is even possible for a lagging player to expend sufficient effort to catch up with his rival and possibly even become the leader again.[5]

We apply our general characterization results to illustrate two additional implications of multi-battle contests. First, individual and aggregate effort, as well as individual probabilities of winning, may be non-monotonic in the closeness of the race. In a battle that takes place in a state of full symmetry, where both players need the same number of component contest wins, players exert the most effort. They compete for both the intermediate prize and the final prize at this state. Both prizes are fully dissipated in expectation in this component contest, due to the cut-throat nature of this competition. If, instead, the multi-battle contest has moved away from this state of perfect symmetry,

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[1] The multi-battle contests that we examine in this article are a special case of what Harris and Vickers (1985, 1987) call a *race*. According to Harris and Vickers (1987, p. 1) a race is "a competition in which a prize is awarded to the first competitor to achieve a given amount of progress." In Harris and Vickers (1985), players alternate in expending resources in order to move their position a certain distance, with the first player to succeed winning a prize. This is a game with complete and perfect information and does not fit our definition of a multi-battle contest. Other models of patent races that do not fit our definition of a multi-battle contest include Fudenberg et al. (1983), Leininger (1991), and Budd et al. (1993).

[2] For a survey of the theory of contests in sports see Szymanski (2003), where the fact that many different battles interact in sports is acknowledged.

[3] Nalebuff and Stiglitz (1983, p. 41) cover this structure as a special case. Dasgupta (1986) applies a one-shot all-pay auction to model a patent race. In this article we argue that under certain conditions it is natural to consider dynamic games such as patent races, sports championship series, or sequential elections as multi-battle contests with component contests that are all-pay auctions.

[4] These include Baye et al. (1996), Clark and Riis (2000), Ellingsen (1991), Hillman and Riley (1989) and Moldovanu and Sela (2001, 2006).

[5] As suspense is one of the desirable features of sports events (see, e.g., Hoehn and Szymanski, 1999), this result may explain why such intermediate prizes are frequently observed in races which are carefully designed. In Formula I races, for instance, each Grand Prix generates some benefits to the winner, apart from the championship points that count for the overall championship that is awarded on an annual basis. Similarly, the PGA tour has large purses of prize money in the various tournaments, but each victory also contributes to the grand prize which is awarded at the end of the tour.