Available online at www.sciencedirect.com
ScienceDirect
Games and Economic Behavior 59 (2007) 1–16
GAMES and Economic Behavior
www.elsevier.com/locate/geb

A random matching theory ☆

C.D. Aliprantis [a], G. Camera [a,*], D. Puzzello [b]

[a] *Department of Economics, Purdue University, West Lafayette, IN 47907-2056, USA*
[b] *Departments of Economics and Mathematics, University of Kentucky, Lexington, KY 40506-0034, USA*

Received 9 February 2006

Available online 11 September 2006

Abstract

We develop theoretical underpinnings of pairwise random matching processes. We formalize the mechanics of matching, and study the links between properties of the different processes and trade frictions. A particular emphasis is placed on providing a mapping between matching technologies and informational constraints.

JEL classification: C00; C78; D83; E00

Keywords: Random matching; Frictions; Spatial interactions

1. Introduction

A large segment of the economic literature is concerned with the study of allocations that arise when markets are not well-functioning. A defining characteristic of this literature is its focus on informational and spatial frictions, and the desire to make them explicit by assuming that economic interactions occur in small coalitions. To this end, the literature has traditionally relied on pairwise random matching frameworks. This basic modeling tool has found use in a wide variety of settings, from the study of social norms (as in Kandori, 1992), to unemployment (as in Mortensen and Pissarides, 1994), to business cycles (as in Diamond and Fudenberg, 1989), and

☆ This research is supported in part by the NSF grants SES-0128039 and DMS-0437210.
* Corresponding author.
E-mail addresses: aliprantis@mgmt.purdue.edu (C.D. Aliprantis), gcamera@mgmt.purdue.edu (G. Camera), daniela.puzzello@uky.edu (D. Puzzello).

0899-8256/$ – see front matter
doi:10.1016/j.geb.2006.08.001

to the foundations of monetary theory (as in Kiyotaki and Wright, 1989; Shi, 1997; and Green and Zhou, 2002).

A limitation of this literature is that the treatment of matching—as a technology—is mostly descriptive and insufficiently formalized. For example, the mechanics of the economic interactions are generally not made explicit or the map between matching and the frictions assumed to be in place is open to various interpretations. This tends to prevent a clear understanding of how the matching technology impairs market functioning, and consequently the possible allocations. These limitations must be overcome to better formulate models of economies with frictions. An objective economic analysis is thought of as one that focuses on the allocations predicted using a carefully specified physical environment (preferences, technologies, etc.). Thus, a comprehensive theory of exchange cannot be derived by simply assuming that certain economic interactions may or may not take place. Ideally, the theory should clarify how the trading or institutional constraints assumed to be in place originate in the underlying economic environment.

The purpose of this study is to build a more solid foundation for random matching models, by means of a set-theoretic approach. There are two major contributions. First, the paper provides a formalization of the *mechanics* of random pairwise matching. To do so, it uses as a starting point the approach to deterministic matching provided by Aliprantis et al. (2006). Compared to that paper, this study introduces the concept of spatial separation in terms of population partitions, and uses probability measures to match agents only within the same partition set. This provides a clear and simple formalization of random matching. By focusing on the technological aspects of meeting processes, this study adds to a literature on matching models.[1]

A second contribution of this investigation is that it spells out how different matching technologies may facilitate (or obstacle) the exchange of economic resources and information among agents. Particular emphasis is paid to formalizing how the matching technology's properties affect the level of informational isolation that exists in economies where agents are randomly paired over time.[2] Indeed, this is what especially differentiates our work from previous studies on random matching processes; see, for instance, the matching scheme of Boylan (1992) for a countable population.

The technical procedure that we use to construct any random matching process involves three basic steps. The first step is to specify how to divide the population in each period into spatially separated clusters of agents. To do so, we use partitional correspondences. Then, one must define and calculate all possible ways to form pairs in each cluster. In this case, we resort to using a class of permutation functions, the so-called involutions. Finally, for each period one must specify a probability measure over all possible pairings, for each cluster. This gives us the desired random matching rule for a cluster, and a well-defined random matching process for the entire population in each period. A pairwise random matching framework can then be formalized as a sequence of partitional correspondences, involutions and probability measures. Given these sequences, we can then explicitly specify matching histories, and therefore we can formalize the degree of informational isolation that exists among agents.

The paper is organized as follows. Section 2 introduces the mathematical background. Sections 3 and 4 discuss pairwise random matching in a single period and over time and characterize

[1] For example, see Ioannides (1990), Gilboa and Matsui (1992), and the more recent works of Duffie and Sun (in press, 2004) on the exact law of large numbers for random pairwise matching.

[2] Research that has taken into consideration these concerns has appeared especially in the monetary literature. For instance, see the works of Huggett and Krasa (1996), Kocherlakota (1998), and Corbae and Ritter (2004).