Available online at www.sciencedirect.com
ScienceDirect
Games and Economic Behavior 58 (2007) 172–180
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Collective choice with endogenous reference outcome

Hannu Vartiainen

Yrjö Jahnsson Foundation, Ludviginkatu 3-5, 00130 Helsinki, Finland

Received 21 April 2003
Available online 7 February 2006

Abstract

A *collective choice* problem—essentially a bargaining problem without disagreement outcome—is studied. An extended solution, which determines a solution and a reference point simultaneously, is characterized. The unique extended solution that meets the extended versions of Pareto-optimality, independence of irrelevant alternatives, symmetry, and scale invariance maximizes the Nash product with respect to *both* the solution *and* the reference point.

JEL classification: C71; C78

Keywords: No disagreement; Bargaining theory; Axioms

1. Introduction

We analyze a two-player collective choice problem, represented by a two dimensional, convex, and compact utility possibility set U. Set U is the only primitive of the model, there is *no* disagreement outcome á la Nash (1950).

In many bargaining scenarios disagreement has a well defined meaning and clear welfare implications. In such cases bargaining theory is the natural framework for analyzing collective decision making. However, in other scenarios it may not be clear what disagreement means: disagreement payoffs may not be part of the data that describes the problem.

Imagine two roommates occupying the only available apartment in town, and agreeing on the rules of the household: when and by whom to clean, cook, listen to music, invite friends, etc. On the one hand, the agreed rule must be self-enforcing, i.e. a (correlated) equilibrium of the underlying game. On the other hand, any such rule can be agreed upon. But even if an agreement

E-mail address: hannu.vartiainen@yjs.fi.

0899-8256/$ – see front matter
doi:10.1016/j.geb.2005.12.005

is not reached, the roommates still have to live together and play the underlying game. Hence, *some* equilibrium will be played even in the absence of an agreement.

To apply bargaining theory to the roommates' negotiation problem, we must know the disagreement payoffs. But if no agreement is reached, then we only know that *some* equilibrium is played. Hence, bargaining theory is of limited use. A theory which does not rely on the existence of a disagreement outcome, or generates one *endogenously,* would solve the problem. Coming up with such a theory is the aim of this paper.

In the arbitration literature, which sees a bargaining solution as a scheme implemented by an impartial and "fair" arbitrator (see e.g. Luce and Raiffa, 1992, and Border and Segal, 1997), the question of how to select the disagreement point is central. Since a typical solution is sensitive to the position of the disagreement point, the success of the arbitration process depends on how this question is solved.[1] However, in this literature, no explanation is typically given as to *why* the chosen disagreement point should be implemented in the absence of agreement.[2] In our view, this calls for an axiomatic treatment.

We analyze the collective choice through an *extended solution* F which specifies, under any collective choice problem U, a nonempty, closed, irreflexive subset of $U \times U$. An element of the extended solution $F(U)$, say (s, r), is an ordered pair of utility vectors where the first entry, s, is referred to as the *solution* and the second, r, as the *reference outcome*.

The solution should be viewed as an acceptable compromise given that in the absence of cooperation players would end up to the reference outcome. Negotiation, in our framework, could be thought as a process that reduces the size of the collective choice problem to a form where an indisputable decision can be made. The binary choice problem between the solution and the reference outcome could be viewed as the final stage of the negotiation process.

Technically, why should we not focus directly on the solution since that is what we are primarily interested in? The problem is that reasonable axiomatizations tend to be too strong to allow any solution—as has become painfully clear in the social choice literature—or too loose to pin down a well defined solution. A reference outcome gives a necessary degree of freedom that permits us to circumvent this problem. The key property of a reference outcome is that it can be used to compare utilities across outcomes. Hence, with a reference outcome the collective choice problem moves from the preference aggregation domain to the utility comparison domain. As argued by Conley et al. (1997, 2000) and Thomson (1981), this difference is crucial (see Section 5 for further discussion).[3]

The axioms we impose on the extended solution are analogues to those of Nash (1950), i.e. *Pareto-optimality, symmetry*, *independence of irrelevant alternatives* (IIA), and *scale invariance*. As there is no fixed disagreement point, also the axioms on the extended solution need to be extended. The extended axioms now assume that the selection process of the *combination* of a solution and a reference point is governed by unified principles.

Our result is the following: The unique extended solution meeting the axioms contains (s, r) under U if and only if (s, r) maximizes the Nash product on U with respect to *s and r*. Such

[1] A related question is analyzed by Border and Segal (1997). They keep the disagreement point fixed but axiomatize the preferences of an arbitrator over possible bargaining solutions. Their analysis gives support to the Nash solution.

[2] In particular, Kalai and Rosenthal (1978) (see also Rosenthal, 1978) allow players to choose the disagreement point noncooperatively. They show that, for a fixed, well behaved arbitration scheme, the induced game has a unique equilibrium outcome.

[3] Conley et al. (2000) (see also Sen, 1982, and Myerson, 1978) show that no social choice rule satisfies the standard axioms of bargaining theory.