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Games and Economic Behavior 56 (2006) 148–173
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Aggregating disparate estimates of chance

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Received 21 October 2003

Abstract

We consider a panel of experts asked to assign probabilities to events, both logically simple and complex. The events evaluated by different experts are based on overlapping sets of variables but may otherwise be distinct. The union of all the judgments will likely be probabilistically incoherent. We address the problem of revising the probability estimates of the panel so as to produce a coherent set that best represents the group's expertise.

JEL classification: C42; C44; C65; C81; C91; C92

Keywords: Aggregation; Probability; Forecasting; Coherence

1. Introduction

1.1. Aggregating opinion

When deciding what to believe, it often makes sense to consult other people's opinions. To be useful, however, the opinions must be processed by some method for converting them into your ultimate judgment. Let us call such a method an *aggregation principle*. Formulating and justifying principles of aggregation pose unexpected challenges. Consider an example raised by

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0899-8256/$ – see front matter
doi:10.1016/j.geb.2006.04.001

Kornhauser and Sager (1986).[1] Three experts issue opinions about the four statements symbolized at the top of the following table.

Expert	p	q	$r \leftrightarrow (p \wedge q)$	r
1	agree	agree	agree	agree
2	agree	disagree	agree	disagree
3	disagree	agree	agree	disagree
Majority	agree	agree	agree	disagree

(1)

Each expert is logically consistent, and you would like to agree with a given statement if and only if the majority of experts do. But this turns out to be unadvisable, since the majority-opinion (shown at the bottom of the table) is logically inconsistent. An alternative formulation of the puzzle distinguishes the "premises" p, q, r from the "conclusion" $r \leftrightarrow (p \wedge q)$ Voting on the premises unexpectedly gives a different answer than voting on the conclusion.[2] The difficulty does not stem from the absence of one's own opinion when trying to integrate those of the experts. You can imagine that you are one of the experts, and even that your opinions are endowed with extra credibility. It is easy to see that a weighted voting scheme will nonetheless come to grief in certain cases.

A more promising response to Kornhauser and Sager's puzzle is to distinguish statements in terms of logical complexity. The experts can be polled just on p, q, r, leaving the status of $r \leftrightarrow (p \wedge q)$ to be deduced. Applied to the table, this policy yields agreement with p, q and disagreement with $r, r \leftrightarrow (p \wedge q)$, which is a consistent point of view. To generalize this approach, a few definitions are necessary.

By a *polarity variant* of a set $\{\varphi_1 \cdots \varphi_m\}$ of formulas we mean any set of form $\{\pm\varphi_1 \cdots \pm\varphi_m\}$, where $\pm\varphi_i$ is φ_i with one or zero occurrences of $\neg$ in front it. For example, $\{p, \neg(q \vee r)\}$ is a polarity variant of $\{p, (q \vee r)\}$. By a *basis* of a set A of formulas is meant a subset B of A that meets the following conditions.

(a) The members of B are *logically independent*, that is, every polarity variant of B is consistent.
(b) For every polarity variant B' of B and formula $\psi \in A$, either B' logically implies ψ or B' logically implies $\neg\psi$.

Intuitively, a basis of A is a minimal subset of A whose truth values suffice to pin down the truth values of the rest of A. For example, $\{p, q\}$ is a basis for $\{p, q, p \wedge q\}$. In contrast, the set $\{p \wedge q\}$ is not a basis for $\{p, q, p \wedge q\}$ because the polarity variant $\{\neg(p \wedge q)\}$ does not imply either p or $\neg p$ (and similarly for q). Thus, a promising strategy for aggregating multiple opinions about a given set of statements is to apply majority rule to some basis for the set, obtaining one of its polarity variants; the rest can be filled in via deduction. The resulting judgments will be consistent.

[1] It is generalized in List and Pettit (2002). Other studies of aggregation within the same framework have produced remarkable findings. See List (2003) and especially Dietrich and List (2004). Discussion of the repercussions of such findings for political theory is offered in Pettit (2001).

[2] The alternative formulation was pointed out to us by a referee. Note that in this example, it is equally natural to think of $r \leftrightarrow (p \wedge q)$ as premise and r as conclusion.