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# Allocation problems among sharing groups

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## Abstract

We deal with allocation problems among sharing groups. There are  $n$  agents. The agents are divided into several sharing groups. A homogeneous good is allocated among sharing groups rather than among the agents. The good is a private good for sharing groups, and a public good for the members of each sharing group. That is, all of them in the same sharing group can consume it without rivalry. We introduce some allocation rules and axioms. The utilitarian allocation rule and the egalitarian allocation rule are characterized by some axioms.

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## 1. Introduction

Consider the following situation. A university has to divide its annual budget among departments. Each department consists of faculty and students. Some of these individuals may belong to several departments. Each department supplies services to its members from its allocated budget. The individuals benefit from the services of all departments to which they belong. How should the university allocate its budget among the departments?

This is the problem of allocation among sharing groups introduced by Lerner (1998). Formally, there are  $n$  agents. The agents are divided into several sharing groups. These

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groups are given exogenously. They are not necessarily disjoint. A homogeneous good is allocated among the groups. The good has two features. It is a private good to the groups, and a public good to the members of the same group. That is, a group cannot use the good allocated to the other group, but all members of the same group have access to the amount allocated to the sharing group as if it were allocated to him alone. The payoff of an agent is the sum of the amounts he consumes in all groups to which he belongs.

Lerner studies this problem from both the cooperative and non-cooperative points of view. First, he formalizes it as a bargaining problem. The bargaining set is the comprehensive hull of the set of feasible payoff vectors of the players. The disagreement point is the zero allocation. He applies the Nash solution (Nash, 1950) to this bargaining problem.

Second, he builds a simple non-cooperative game. Each player's strategy is an allocation of the good among sharing groups. The outcome of the game is the average of the players' strategies. He presents a criterion, called "disappointment-freeness,"<sup>1</sup> for selecting an equilibrium from the set of Nash equilibria. He shows that there is only one equilibrium satisfying this criterion, and that the corresponding outcome is the Nash solution outcome of the bargaining problem.

In this paper, we analyze the problem from a different perspective. We follow the axiomatic approach. First, we introduce a number of allocation rules. An allocation rule is a function that assigns each problem a group allocation. A group allocation indicates the amounts received by groups. The "egalitarian" rule gives all groups equal amounts. The "utilitarian" rule<sup>2</sup> allocates the good to groups so as to maximize the sum of individual payoffs.<sup>3</sup>

We also introduce a number of axioms. *Pareto efficiency* requires that the allocation rule should allocate the good to groups such that "the induced individual allocation" is not Pareto dominated by a feasible individual allocation. *Equal treatment of equal groups* requires that if two sharing groups are equal, then they should receive the same amount. *Equal treatment of equal individuals* requires that if two agents belong to the same number of sharing groups, then they should receive the same amount. *Group monotonicity* requires that if all groups increase the same number of agents, then the amount given to groups should be the same as before. *Group Consistency* requires that if an allocation rule chooses a certain allocation for a problem, then for the reduced problem obtained when some groups leave with their amounts, the allocation rule should assign to the remaining groups the same amounts as before.<sup>4</sup>

We characterize the modified utilitarian rule and the egalitarian rule. The modified utilitarian rule is the unique rule satisfying *Pareto efficiency*, *equal treatment of equal groups*, *group monotonicity*, and *group consistency*. The egalitarian rule is the unique rule satisfy-

<sup>1</sup> Player  $i$  is *disappointed* with a sharing group given a strategy profile if his contribution to the sharing group is greater than the sharing group's contribution to him. A Nash equilibrium is *disappointment-free* if no player is disappointed with any sharing group given this strategy profile.

<sup>2</sup> The utilitarian rule is not single-valued. We modify the rule so as to make it single-valued.

<sup>3</sup> We introduce other rules such as *proportional* rule, *Nash* rule, *maximin* rule and so on.

<sup>4</sup> We also introduce other axioms related to *lower-bounds* and *positivity* properties.

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