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Note

A note on a value with incomplete communication

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Abstract

The Myerson's models on partial cooperation have been studied extensively [SIAM J. Discrete Math. 5 (1992) 305; Math. Methods Operations Res. 2 (1977) 225; Int. J. Game Theory 19 (1980) 421; 20 (1992) 255]. In [Game Econ. Behav. 26 (1999) 565], Hamiache proposes a new solution concept for communication situations. In this work, we analyze this value making some deficiencies clear and generalize this value to union stable cooperation structures emphasizing the differences in the extension.

JEL classification: C71

Keywords: Cooperative game; Communication situation; Consistency

1. Introduction

In the general model of cooperative games it is assumed that there is no restriction in the formation of coalitions and so, a *cooperative game* with transferable utility is defined as a pair (N, v), where N is a finite set of players and $v : 2^N \to \mathbb{R}$ is a function that assigns to each $S \subseteq N$ a worth $v(S)$ and verifies that $v(\emptyset) = 0$. However, in many practical situations the cooperation is not complete. The *communication situations* model (Myerson, 1977; Owen, 1986), where the relationships among the players are represented by undirected

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graphs, is one of the most important. The restricted game by a graph (N, G) is defined by $v^G : 2^N \to \mathbb{R}$, $v^G(S) = \sum_{T \in S/G} v(T)$ where S/G is the set of connected components of $S \subseteq N$. The most extensively studied solution concepts in communication situations have been the Shapley value of the restricted game or Myerson (1977) value, and the position value (Meesen, 1988; Borm et al., 1992) which discriminates the value of each player by defining a game over the edges of the graph.

In the paper "A Value with Incomplete Communication," Hamiache (1999) makes a critical valuation of the Myerson value and the position value in communication situations and concludes that these values do not discriminate the players enough according to their position in the graph. He proposes a new solution concept for communication situations and gives an axiomatic characterization based mainly on the notions of associated game and consistency.

In this work, we analyze this value and indicate some deficiencies in the results. Moreover, we generalize this value to *union stable cooperation structures,* which have communication situations as a particular case. The justification of union stable structures comes from Myerson himself who pointed out the limitations of communication situations and modeled the relationships among the players by means of hypergraphs (Myerson, 1980). Later, van den Nouweland et al. (1992), Slikker and van den Nouweland (2001) studied these structures through communication hypergraphs.

2. A value with incomplete communication

In this section, we resume the Hamiache's results. We denote a communication situation by (N, v, G) where (N, v) is a cooperative game and (N, G) is a graph and by SC^N the set of all communication situations on N. Given a graph (N, G) and $S \subseteq N$, let $S^* = \{i \in N\colon \exists j \in S \text{ such that } \{i, j\} \in G\}$.

If ϕ is a solution on SC^N, for all (N, v, G), its associated game (N, v^*_ϕ, G) is defined, for $S \subseteq N$, by

$$v^*_\phi(S) = \begin{cases} v(S) + \sum_{j \in S^* \setminus S} \left[\phi_j\left(S \cup \{j\}, v_{S \cup \{j\}}, G(S \cup \{j\})\right) - v(\{j\})\right] \\ \quad \text{if } S \text{ connected}, \\ \sum_{T \in S/G} v^*_\phi(T) \quad \text{otherwise}, \end{cases}$$

where $(T, v_T, G(T))$ is the communication situation restricted to the coalition T.

Hamiache formulates the following axioms, where u_R is the unanimity game corresponding to the coalition R, that is, for $S \subseteq N$,

$$u_R(S) = \begin{cases} 1 & \text{if } R \subseteq S, \\ 0 & \text{otherwise.} \end{cases}$$

H1 Component-efficiency. For (N, v, G) and $S \in N/G$, $\sum_{j \in S} \phi_j(N, v, G) = v(S)$.

H2 Linearity with respect to the game. For all $\alpha, \beta \in \mathbb{R}$ and (N, v, G), $(N, w, G) \in SC^N$, $\phi(N, \alpha v + \beta w, G) = \alpha\phi(N, v, G) + \beta\phi(N, w, G)$.

H3 Independence of irrelevant players. For all (N, G), for all connected coalitions R, T with $R \subseteq T$ and for $i \in T$, $\phi_i(N, u_R, G) = \phi_i(T, u_R, G(T))$.