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An efficient finite volume model for shallow geothermal systems. Part I: Model formulation

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ABSTRACT

This series of two papers presents a three-dimensional finite volume model for shallow geothermal systems. In this part, an efficient computational model describing heat and fluid flow in ground-source heat pumps is formulated. The physical system is decomposed into two subdomains, one representing a soil mass, and another representing one or a set of borehole heat exchangers. Optimization of the computational procedure has been achieved by, first, using a pseudo three-dimensional line element for modeling the borehole heat exchanger, and second, using a combination of a locally refined Cartesian grid and a multigrid with hierarchal tree data structure for discretizing and solving the soil mass governing equations. This optimization made the model computationally efficient and capable of simulating multiple borehole heat exchangers embedded in a multilayer system, in relatively short CPU time. In Part II of this series, verifications and numerical examples describing the computational capabilities of the model are presented.

1. Introduction

Geothermal heat is a renewable source of energy generated in the core of the earth, about 6000 km below the surface. Geothermal energy offers a number of advantages over conventional fossil-fuel resources, particularly the geothermal heat resources are locally available, renewable, economic, and their environmental impact in terms of CO_2 emissions is significantly lower. Deep geothermal resources have been exploited for the production of electricity and other direct uses (Bertani, 2005). Shallow geothermal resources have been widely used for direct uses, mainly space and greenhouse heating purposes (Lund et al., 2005). In this contribution, focus is placed on shallow geothermal systems, mainly the ground-source heat pumps.

Currently, different numerical methods have been utilized for the simulation of shallow geothermal systems. The finite difference method seems to dominate this research field, as it is traditionally utilized to solve heat flow problems. It is employed by Eskilson and Claesson (1988), Clauser (2002), Sliwa and Gonet (2005), Lee and Lam (2008), among others. The finite element method is recently gaining momentum and seems to be first employed by Muraya (1994), followed by, among others, Al-Khoury et al. (2005, 2010), Al-Khoury and Bonnier (2006), Marcotte and Pasquier (2008), Diersch et al. (2011a,b) and Raymond et al. (2011). The finite volume method has also been utilized to model ground-source heat exchangers. One of the earlier works in this field is that by Yavuzturk (1999), who developed a two-dimensional, fully implicit finite volume model using an automated parametric grid generation algorithm for different pipe sizes, shank spacing and borehole geometry. Rees et al. (2004) used the finite volume method to develop an axial-symmetric BHE-soil with groundwater flow model. This model was later extended and utilized by He et al. (2011) to simulate three-dimensional ground-source heat pumps. A multi-block structured mesh is used in this model, allowing the simulation of complex geometry around the borehole heat exchangers (BHE).

Most efforts for developing numerical models for shallow geothermal systems are spent on tackling two main issues: disproportional geometry and heat convection. Shallow geothermal systems, particularly vertical ground-source heat pumps, consist of very slender borehole heat exchangers (BHE) embedded in a vast soil mass. This geometrical peculiarity exerts enormous computational burden, as a combination of very fine elements (cells) and coarse elements (cells) is normally needed to discretize the computational domain. For three-dimensional systems, this normally requires hundreds of thousands to millions of elements, making the CPU time unrealistic for engineering purposes. This problem exuberates in the presence of convection and groundwater flow. Governing equations of cases with relatively high Peclet numbers behave like hyperbolic functions, which require fine meshes (grids) and proper upwind schemes. Different solutions have been proposed in literature for tackling these

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problems. Here, we utilize the numerical model proposed by Al-Khoury et al. (2005), Al-Khoury and Bonnier, (2006).

In this contribution, a three-dimensional finite volume model for heat and fluid flow in ground-source heat pumps is formulated. A structured locally refined grid is utilized for the discretization of the spatial domain, and the theta method for the discretization of the temporal domain. The physical model is decomposed into two subdomains, one representing the soil mass, and the other representing one or more borehole heat exchangers. The soil mass is simulated as a fully saturated porous medium with groundwater flow, and the BHE is modeled using a pseudo three-dimensional line element. The thermal interaction between the BHE and soil mass is calculated sequentially by considering each one of them as a heat source to the other one, located at the contact surface between them.

In the following, governing equations representing the initial and boundary value problem of typical ground-source heat pumps are given. Then, a detailed formulation of the corresponding finite volume model is described. Later, a solution algorithm is outlined.

2. Governing equations

Governing equations of a shallow geothermal system, in particular, a ground-source heat pump, comprise heat equations and initial and boundary conditions of a soil mass and a borehole heat exchanger. The governing equations of the soil mass include also the continuity equation of the groundwater flow.

2.1. Soil heat and continuity equations

Heat transfer in a shallow geothermal system arises from the thermal interaction between the borehole heat exchangers and the soil mass. In a one-phase soil mass, constituting a solid skeleton, heat is merely conductive, while in the presence of groundwater flow, the soil mass constitutes a multiphase porous medium and the heat flow in this case is conductive-convective. In typical shallow geothermal systems, it is reasonable to assume that the groundwater flow is steady-state, and occurs in confined fully saturated soil layers. Furthermore, the groundwater flow is independent of temperature, but the temperature of the soil is highly dependent on the groundwater flow.

In the presence of groundwater, the transient heat flow balance equation of a soil mass can be described as

$$\rho c \frac{\partial T}{\partial t} = \nabla(\boldsymbol{\lambda}\nabla T) - \nabla(\rho_w c_w \mathbf{v} T) + H(x,y,z) \tag{1}$$

in which the subscript w refers to water, ρc (J/m^3 K) is the volumetric heat capacity, with ρ (kg/m^3) the mass density and c (J/kg K) the specific heat capacity, $\boldsymbol{\lambda}$ (W/m K) is the thermal conductivity tensor, $\mathbf{v}$ (m/s) is the Darcy velocity vector and $H(x,y,z)$ is a heat source. In practice, $\boldsymbol{\lambda} = \lambda_x = \lambda_y = \lambda_z = \lambda$, and for a two-phase material, the thermal conductivity and the volumetric heat capacity are described in terms of a local volume average, as

$$\begin{aligned} \lambda &= (1-n)\lambda_s + n\lambda_w \\ \rho c &= (1-n)\rho c_s + n\rho c_w \end{aligned} \tag{2}$$

where the subscript s refers to solid, and n is the material porosity. The Darcy velocity vector can be described as

$$\mathbf{v} = -\mathbf{k}\,\nabla\varphi \tag{3}$$

in which $\mathbf{k} = (\tilde{\mathbf{k}}\rho g/\mu)$ is the hydraulic conductivity tensor, commonly termed permeability (m/s), where $\tilde{\mathbf{k}}$ (m^2) is the intrinsic permeability tensor, μ (kg/ms) is the dynamic viscosity, and g (m/s^2) is the gravity. In engineering practice, $\boldsymbol{k} = k_x = k_y = k_z = k$.

φ (m) in Eq. (3) is the total head, defined as

$$\varphi = \frac{P}{\rho g} + z \tag{4}$$

where $P/\rho g$ is the pressure head and z is the elevation head. The fluid flow balance equation, describing groundwater flow, is expressed as

$$S\frac{\partial \varphi}{\partial t} = -\nabla(\mathbf{k}\varphi) + Q(x,y,z) \tag{5}$$

in which S (m^{-1}) is the specific storage coefficient, which represents the amount of water released per unit area per head gradient and $Q(x,y,z)$ is a source or a sink.

2.1.1. Soil initial and boundary conditions

For the fluid flow, the initial condition in the soil mass, at time $t=0$, is described in terms of the hydrostatic head as

$$\varphi(x,y,z,0) = \varphi_0(x,y,z) \tag{6}$$

The boundary conditions for groundwater flow is most likely associated with a head difference between an upper stream and a lower stream occurring in a confined aquifer, described as

$$\begin{aligned} &\varphi(x=0,\,y,\,z) = \varphi_1, && \text{on } x=0 \text{ surface} \\ &\varphi(x=L,\,y,\,z) = \varphi_2, && \text{on } x=L \text{ surface} \\ &\mathbf{k}\nabla\varphi\,\mathbf{n} = J, && \text{on any of the boundary surfaces} \end{aligned} \tag{7}$$

in which L is the length (along the x-axis) of the distance between two known hydraulic heads, and J is a fluid flux.

For the heat flow, the initial condition of the soil mass, at time $t=0$, is defined as the steady-state condition

$$T(x,\,y,\,z,\,0) = f(x,\,y,\,z) \tag{8}$$

Boundary conditions associated with a shallow geothermal field might be:

$$\begin{aligned} &T(\mathbf{x},t) = f(\mathbf{x},t), && \text{on a point or a surface } \boldsymbol{x} \\ &\boldsymbol{\lambda}\nabla T\mathbf{n} = b_{as}\,(T_s - T_a), && \text{on the surface in contact with the air} \\ &\boldsymbol{\lambda}\frac{\partial T}{\partial n} = b_{gs}(T_s - T_g), && \text{on the surface in contact with a BHE} \end{aligned} \tag{9}$$

in which T_a is the air temperature, b_{as} is the convective heat transfer coefficient at the surface in contact with the air, T_g is the pipe (grout) temperature and b_{gs} is the reciprocal of the thermal resistance between the soil and the grout (borehole).

2.2. Borehole heat exchanger heat equations

Heat transfer in a borehole heat exchanger is conductive-convective and arises from the flow of a working fluid (refrigerant) circulating the pipes, and the thermal interaction between the borehole components and the surrounding soil mass. Heat transfer equations of a typical single U-tube BHE consisting of a pipe-in (denoted as i), a pipe-out (denoted as o), and a grout (denoted as g) can be described as (Al-Khoury, 2006)

Pipe-in

$$\rho c_r \frac{\partial T_i}{\partial t} - \lambda_r \frac{\partial^2 T_i}{\partial z^2} + \rho c_r u \frac{\partial T_i}{\partial z} = b_{ig}(T_g - T_i) \tag{10}$$

Pipe-out

$$\rho c_r \frac{\partial T_o}{\partial t} - \lambda_r \frac{\partial^2 T_o}{\partial z^2} - \rho c_r u \frac{\partial T_o}{\partial z} = b_{og}(T_g - T_o) \tag{11}$$

Grout

$$\rho c_g \frac{\partial T_g}{\partial t} - \lambda_g \frac{\partial^2 T_g}{\partial z^2} = b_{ig}(T_i - T_g) + b_{og}(T_o - T_g) + b_{gs}(T_s - T_g) \tag{12}$$