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Symplectic partitioned Runge–Kutta methods for two-dimensional numerical model of ground penetrating radar

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ABSTRACT

Simulation of ground penetrating radar (GPR) wave propagation in two-dimensional (2-D) subsurface structure is developed using the symplectic partitioned Runge–Kutta (SPRK) method. A transmitting boundary is implemented to absorb waves at the edges of the modeling. For the 2-D case, the SPRK schemes require only two functions for the complete description of the electromagnetic field. To verify the performance of the proposed algorithms, results from comparisons between the SPRK schemes and the standard FDTD method are presented. In addition, a complicated subsurface structure model is considered. The wiggle trace profile of this model is obtained from forward simulation using a 2-order SPRK method.

1. Introduction

Ground penetrating radar (GPR), as a kind of high resolution, non-destructive tool, has been widely applied to various disciplines, such as archaeology (Böniger and Tronicke, 2010), civil engineering (Xu et al., 2010), and mine detection (Zyada et al., 2011). A ground-based GPR transmitting antenna generates an electromagnetic pulse and transmits it into the earth. The system then records reflected, scattered, and diffracted events from boundaries of subsurface materials and buried objects in the earth received by an antenna on the surface. The locations of different buried objects and the boundaries of the underground media can be determined from the GPR data. Simulation of the electromagnetic wave propagation in subsurface structure is useful to get a better interpretation of real GPR profiles. It can provide one means of exploring the link between subsurface properties and GPR data.

Various models and approaches have been developed for numerical models of GPR. These include ray-tracing methods (Cai and McMechan, 1995; Zeng et al., 1995), Jonscher models (Hollender and Tillard, 1998; Grégoire and Hollender, 2004), pseudo-spectral methods (Casper and Kung, 1996; Carcione et al., 1999), integral methods (Xiong and Tripp, 1997), Fourier methods (Bitri and Grandjean, 1998; Bano, 2004; Ghasemi et al., 2007), the finite difference time domain (FDTD) technique (Bergmann et al., 1996; Roberts and Daniels, 1997; Chen and Hang, 1998; Irving and Knight, 2006; Çakir and Sevgi, 2009), and ADI-FDTD technique (Diamanti and Giannopoulos, 2009; Feng and Dai, 2011). Unfortunately, these techniques and models are limited in their engineering applications due to some disadvantages. For instance, the ray-tracing method is difficult for modeling complicated structure (Zeng et al., 1995), the FDTD method needs large memory and significant calculation time to achieve high precision because of the restriction of the Courant–Friedrichs–Lewy (CFL) stability condition (Taflove, 1995), and the ADI-FDTD method eliminates the CFL limit successfully, but a large time step also increases dispersion errors (Zheng and Chen, 2001).

It is well known that situations where dissipation is not significant can be modeled by Hamiltonian systems of ordinary, or partial, differential equations. The symplectic schemes are designed to preserve the global symplectic structure of the phase space for a Hamiltonian system. By symplectic we mean conservation of energy. They show substantial benefits in numerical computation for a Hamiltonian system, especially in long-term simulations. Recently, the symplectic schemes have been implemented in calculating the electromagnetic fields in lossless media. Hirono et al. (1997) developed a high-order symplectic integrator for the 2-D time-domain simulation of the electromagnetic field. Kusaf et al. (2005) presented a new scheme for the calculation of the coefficients of the exponential differential operators of the

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symplectic FDTD method. A four-stage optimized symplectic integrator propagator (Huang et al., 2006) was developed for the 3D electromagnetic scattering problems. Sha et al. (2007) proposed an explicit fourth-order symplectic FDTD scheme to electromagnetic simulation in 3-D. However, because the dissipation of most underground materials can not be ignored, the problem of GPR propagation in underground structures can not be seen as the classical Hamiltonian system. Fortunately, the symplectic algorithms can still be used for simulation of the electromagnetic wave propagation in lossy media (Sun, 1997).

The symplectic partitioned Runge–Kutta methods (Liu and Sun, 2004; Jiang et al., 2006; Huang and Wu, 2006) are a class of symplectic schemes, which include a variety of different time discretization schemes to preserve the symplectic structure. Among these schemes, the 1-order Radau IA-I $\overline{A}$ and 2-order Lobatto IIIA-IIIB SPRK methods are suitable for simulation of the GPR wave propagation in lossy media because both of these two schemes are explicit. Moreover, for 2-D case, the SPRK methods require only two functions to evolve per time step; in the standard FDTD method, three are needed. Hence, the SPRK schemes can save computer memory usage and CPU time significantly.

In this paper, simulations of ground penetrating radar (GPR) wave propagation in two-dimensional (2-D) subsurface structure are developed using 1-order Radau IA-I $\overline{A}$ and 2-order Lobatto IIIA-IIIB SPRK methods, respectively. The transmitting boundary is adopted to avoid reflections from the edges of the modeling. To begin, we present the Hamiltonian system and the SPRK methods, including the governing equations and their finite-difference approximations. Next, the boundary condition and numerical stability are discussed briefly. Finally, we present two examples, one showing the comparison between the SPRK methods and the standard FDTD scheme, and the other one showing simulation of GPR wave propagation in the complicated subsurface structure.

2. Theory

2.1. Hamiltonian system and SPRK methods

The Hamiltonian system of canonical equations (Sun, 1995; Huang and Wu, 2006) is given by

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\frac{\partial H}{\partial q_i} \overset{\text{def}}{=} f_i(q,p), \quad \frac{\mathrm{d}q_i}{\mathrm{d}t} = \frac{\partial H}{\partial p_i} \overset{\text{def}}{=} g_i(q,p), \quad i=1,2,\ldots,n \tag{1}$$

where H represents Hamiltonian function $H(q_i,p_i)$ $(i=1,2,\ldots,n)$, the integer n is the number of degrees of freedom. "def" means definition. g and f are dependent variables, such as (E, H) and q and p are independent variables, such as distance and time. Hamiltonian systems often have a special separated structure such that

$$H = H(p,q) = V(q) + U(p). \tag{2}$$

If the Hamiltonian is separable, the canonical equations can take the partitioned form with

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\frac{\partial V}{\partial q_i} \overset{\text{def}}{=} f_i(q), \quad \frac{\mathrm{d}q_i}{\mathrm{d}t} = \frac{\partial U}{\partial p_i} \overset{\text{def}}{=} g_i(p). \quad i=1,2,\ldots,n \tag{3}$$

An s-stage partitioned Runge–Kutta method can be specified by a Butecher tableau (Butcher, 2008)

$$\begin{array}{c|ccc} c_1 & a_{11} & \cdots & a_{1s} \\ \vdots & \vdots & & \vdots \\ c_s & a_{s1} & \cdots & a_{ss} \\ \hline & b_1 & \cdots & b_s \end{array}, \quad \begin{array}{c|ccc} C_1 & A_{11} & \cdots & A_{1s} \\ \vdots & \vdots & & \vdots \\ C_s & A_{s1} & \cdots & A_{ss} \\ \hline & B_1 & \cdots & B_s \end{array}. \tag{4}$$

Applying (4) to the Hamiltonian system (1), the following relations can be obtained.

$$\begin{cases} P_i = p^n + \tau \sum_{j=1}^{s} a_{ij} f(Q_j) \\ Q_i = q^n + \tau \sum_{j=1}^{s} A_{ij} g(P_j) \\ p^{n+1} = p^n + \tau \sum_{i=1}^{s} b_i f(Q_i) \\ q^{n+1} = q^n + \tau \sum_{i=1}^{s} B_i g(P_i), \end{cases} \quad i=1,2,\cdots,s \tag{5}$$

where P_i and Q_i are the internal stages corresponding to the variables p and q.

The s-stage PRK method is sympletic (Sanz-Serna, 1988) if the coefficients satisfy

$$\begin{aligned} & b_i A_{ij} + B_i a_{ji} - b_i B_j = 0 \\ & b_i = B_i \quad i,j = 1,\cdots,s. \end{aligned} \tag{6}$$

The coefficients of 1-order Radau IA-I $\overline{A}$ and 2-order Lobatto IIIA-IIIB SPRK methods are shown to be

$$\begin{array}{c|c} 0 & 1 \\ \hline & 1 \end{array}, \quad \begin{array}{c|c} 0 & 0 \\ \hline & 1 \end{array}. \tag{7}$$

$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1/2 & 1/2 \\ \hline & 1/2 & 1/2 \end{array}, \quad \begin{array}{c|cc} 0 & 1/2 & 0 \\ 1 & 1/2 & 0 \\ \hline & 1/2 & 1/2 \end{array}. \tag{8}$$

2.2. Governing equations

Within isotropic lossy material, the Maxwell equations can be written as

$$\begin{aligned} \frac{\partial \mathbf{E}}{\partial t} &= \frac{1}{\varepsilon} \nabla \times \mathbf{H} - \mathbf{J}, \\ \frac{\partial \mathbf{H}}{\partial t} &= -\frac{1}{\mu} \nabla \times \mathbf{E}, \end{aligned} \tag{9}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field vectors, the current density $\mathbf{J} = \sigma \mathbf{E}$, and ε, μ and σ are dielectric permittivity, permeability and conductivity, respectively.

Letting $\mathbf{H} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -U$, the generalized Hamiltonian function in lossy media become

$$H(\mathbf{A},\mathbf{U}) = \int \left(\frac{1}{2\mu} |\mathbf{U}|^2 + \frac{1}{2\varepsilon} |\nabla \times \mathbf{A}|^2 - \frac{1}{\varepsilon} \mathbf{JA} \right) \mathrm{d}V. \tag{10}$$

Using Eq. (1), the Maxwell equations can be represented as canonical equations of the Hamiltonian, such that

$$\begin{aligned} \frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} &= \frac{\mathrm{d}\mathbf{H}}{\mathrm{d}\mathbf{U}} = \frac{1}{\mu} \mathbf{U}, \\ \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t} &= -\frac{\mathrm{d}\mathbf{H}}{\mathrm{d}\mathbf{A}} = \frac{1}{\varepsilon} \nabla^2 \mathbf{A} - \frac{\sigma}{\varepsilon} \mathbf{U}. \end{aligned} \tag{11}$$

Considering the two-dimensional TM case, Eq. (11) can be expressed as

$$\begin{aligned} \frac{\mathrm{d}\mathbf{A}_z}{\mathrm{d}t} &= \frac{1}{\mu} \mathbf{U}_z, \\ \frac{\mathrm{d}\mathbf{U}_z}{\mathrm{d}t} &= \frac{1}{\varepsilon} \nabla^2 \mathbf{A}_z - \frac{\sigma}{\varepsilon} \mathbf{U}_z, \end{aligned} \tag{12}$$

where $\boldsymbol{A}_z$ and $\boldsymbol{U}_z$ denote the z-components of the vectors $\boldsymbol{A}$ and $\boldsymbol{U}$, respectively.