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WaveAR: A software tool for calculating parameters for water waves with incident and reflected components

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ABSTRACT

The ability to determine wave heights and phases along a spatial domain is vital to understanding a wide range of littoral processes. The software tool presented here employs established Stokes wave theory and sampling methods to calculate parameters for the incident and reflected components of a field of weakly nonlinear waves, monochromatic at first order in wave slope and propagating in one horizontal dimension. The software calculates wave parameters over an entire wave tank and accounts for reflection, weak nonlinearity, and a free second harmonic. Currently, no publicly available program has such functionality. The included MATLAB®-based open source code has also been compiled for Windows®, Mac® and Linux® operating systems. An additional companion program, *VirtualWave*, is included to generate virtual wave fields for *WaveAR*. Together, the programs serve as ideal analysis and teaching tools for laboratory water wave systems.

1. Introduction

Surface wave characteristics are of great importance to a wide range of littoral processes, such as wave attenuation and transformation, sediment transport and bed morphodynamics. In the laboratory, accurately resolving water wave characteristics is paramount for controlling experimental conditions. Despite the growing use of more complex wave spectra (due to advances in wavemaker controls), monochromatic or Stokes waves are still integral to many experiments, such as mine burial studies (Cataño-Lopera and García, 2005, 2007), wave friction factor estimates (Carter, 2002), and bedform evolution (Cataño-Lopera and García, 2006a, b; Hancock et al., 2008; Landry and Garcia, 2007; Landry et al., 2007). For such studies, researchers generally need to resolve and quantify the incident wave amplitude, reflection coefficient, and other key parameters.

Existing methods to calculate wave parameters include the two-wave gage method (Goda and Suzuki, 1976; Thornton and Calhoun, 1972), three-wave gage method (Isaacson, 1991; Mansard and Funke, 1980), and maximum/minimum wave height method. These methods estimate the incident wave amplitude and reflection coefficient based on wave measurements at each gage. In addition, since the three-gage method measures the phase lags between gages, it can also estimate the phase of the reflection coefficient. The maximum/minimum method, though simple, is regarded as time consuming and subject to human error (Nallayarasu et al., 1995). The two- and three-gage methods are error prone due to their sensitivity to frequency range, noise, nonlinear harmonics, and probe spacing. Most problematic, though, is the fact that these methods only employ measurements from up to three fixed positions. In a wave field with incident and reflected components, the wave height is modulated over each wavelength. Thus, the previous methods will ultimately fail to accurately resolve the incident and reflected components unless the gages are accurately positioned with prior knowledge of the wave field.

Beyond accurate first harmonic wave parameters, additional information is often required to understand and model the wave field. For example, complete knowledge of the wave field's spatial variation is crucial to understanding bedform ripple geometries and migration velocities (Landry, 2011; Landry et al., 2009). To properly model sandbar formation under Stokes waves, the first and second wave harmonics and the second order Stokes flow must be resolved, including the free and bound components of the second harmonic (Hancock et al., 2008; Landry et al., 2007). The presence of a pronounced, free second harmonic can significantly alter the surface wave envelope and impact bedform development (Hancock et al., 2008).

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The user-friendly wave analysis software tool presented herein, *WaveAR*, estimates wave parameters based on an arbitrary number of wave gage positions. In general, the more wave gage positions that are distributed along the wave field, the more accurately the wave parameters may be resolved. *WaveAR* was developed as part of a combined experimental and numerical effort to observe, measure, and predict the evolution of water waves over sandy bedforms evolving in response to the waves (Hancock, 2005; Hancock et al., 2008; Landry, 2004; Landry et al., 2007). In the regime of interest, and therefore the regime of validity of our software, water waves are adequately described by second order Stokes wave theory. *WaveAR* resolves the full Stokes wave field to second order in wave slope, including the amplitudes and phases of the first and second harmonic wave components. Parameters are estimated by a least squares fit to the measured wave elevations along the wave flume. The theory underlying *WaveAR* is valid for both low and high reflection coefficients and is a generalization of Rosengaus' (1987) reference measuring method valid for low reflection coefficients (Hancock, 2005; Hancock et al., 2008). *WaveAR* has been continuously improved and validated throughout its development against a host of experimental conditions (Hancock et al., 2008; Landry et al., 2007, 2009; Landry and Garcia, 2007).

In what follows, we give a brief theoretical background that includes the mathematical representation of the various wave harmonics and describes how *WaveAR* resolves the bound and free second wave harmonics. We then take the reader on a tour of the *WaveAR* tool and work through two demonstrative examples, including free second harmonic cancellation.

2. Brief theoretical background

A description of Stokes wave theory is found in most standard wave mechanics texts (Dean and Dalrymple, 1991; Mei, 1989). A piston wavemaker with a monochromatic sinusoidal motion will generate a water wave at the fundamental frequency, a bound second harmonic wave associated with Stokes theory (Madsen, 1971) and a free second harmonic. The bound harmonic is an artifact of nonlinearities at the free surface, while the free second harmonic results from the mismatch between the motion of the vertical wall of the wavemaker piston and the fluid particle orbital displacement. The wavelength mismatch between the bound and free second harmonics modulates the total amplitude of the second harmonic along the wave tank. While the derivation details are left to previous work (Hancock et al., 2008), the formulas used by *WaveAR* to resolve the various wave parameters are listed below. The free surface elevation is described mathematically by

$$\eta(t) = A\,\mathrm{Re}(e^{i(kx-\omega t)} + R\,e^{i(\theta-kx-\omega t)}) + \mathrm{Re}((\eta_{2b}+\eta_{2f})e^{-2i\omega t}) \quad (1)$$

where the complex amplitudes of the bound and free second harmonic components are, respectively,

$$\eta_{2b} = \frac{A^2 k(1+2\cosh^2(kh))\cosh(kh)}{4\sinh^3(kh)}(e^{2ikx} + R^2\,e^{2i(\theta-kx)}) \quad (2)$$

$$\eta_{2f} = A_2\,e^{i\alpha_2}(e^{ik_2x} + R_2\,e^{i(\theta_2-k_2x)}) \quad (3)$$

The total amplitude of the second harmonic is

$$\eta_2 = |\eta_{2b}+\eta_{2f}| \quad (4)$$

where vertical bars denote the modulus of a complex number. All parameters in Eqs. (1)–(3) are defined in Table 1. The wavenumbers are related to the angular frequency ω and water depth h by

Table 1
Wave parameters used in *WaveAR* and *VirtualWave*.

Variable	Description
First harmonic	
A	Incident amplitude
R	Reflection coefficient
θ	Relative phase of reflection coefficient
k	Wavenumber
T	Wave period
ω	Angular frequency
Free second harmonic	
A_2	Incident amplitude
k_2	Wavenumber
R_2	Reflection coefficient
θ_2	Relative phase of reflection coefficient R_2
α_2	Relative phase of incident amplitude A_2
Other parameters	
x	Longitudinal distance from the wavemaker
t	Time
h	Still water depth
L	Wave amplitude attenuation coefficient

the dispersion relations

$$\omega^2 = gk\tanh(kh), \quad (2\omega)^2 = gk_2\tanh(k_2h) \quad (5)$$

The wavelengths are given by $\lambda=2\pi/k$ and $\lambda_2=2\pi/k_2$. In general, $\lambda_2 \neq 2\lambda$, as noted above, leading to second harmonic modulation over many wavelengths. The envelope of the first harmonic is

$$\eta_1 = A\sqrt{1+R^2+2R\cos(2kx-\theta)} \quad (6)$$

and the rms amplitude of the Stokes wave is

$$\eta_{\mathrm{rms}} = \sqrt{\frac{\eta_1^2+\eta_2^2}{2}} \quad (7)$$

WaveAR performs Fourier decompositions of the wave elevation time series measured at fixed positions along the wave tank. The resulting wave harmonic amplitudes are compared to Eqs. (4) and (6) and the parameters in Table 1 are found by least squares fitting. For validation and instruction, the companion tool *VirtualWave* performs the reciprocal calculation: given the wave parameters in Table 1, *VirtualWave* uses Eqs. (1)–(7) to produce wave elevation time series, and harmonic and rms amplitudes. For purely incident waves ($R=0$), the parameter L specifies the wave damping and A is replaced by $A\,e^{-Lkx}$ in both *WaveAR* and *VirtualWave*. Lastly, Eqs. (1)–(6) are technically valid only on a flat bottom; bedforms such as sandbars can alter the spatial variation of A, R, and θ along the tank (Hancock et al., 2008). In fact, the reflection of incident waves from bedforms can lead to an apparent damping of the incident wave (Hancock et al., 2008).

3. Software overview

The software package includes two programs: the main program *WaveAR* and a simple companion program, *VirtualWave*. *WaveAR* is short for "wave amplitude and reflection" and is pronounced "Wave·A·R". The software provides an interactive graphical user interface (GUI) for wave parameters to be fit to measured data. *VirtualWave* generates virtual wave elevations at specified numerical probe locations and is intended to be used as a validation and instruction tool to complement *WaveAR*. The open source MATLAB® files for *WaveAR* and *VirtualWave* are available as Electronic Supplementary Information (ESI) included with this paper. The latest version of the software including