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On the optimality of periodic barrier strategies for a spectrally positive Lévy process

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ABSTRACT

We study the optimal dividend problem in the dual model where dividend payments can only be made at the jump times of an independent Poisson process. In this context, Avanzi et al. (2014) solved the case with i.i.d. hyperexponential jumps; they showed the optimality of a (periodic) barrier strategy where dividends are paid at dividend-decision times if and only if the surplus is above some level. In this paper, we generalize the results for a general spectrally positive Lévy process with additional terminal payoff/penalty at ruin, and also solve the case with classical bail-outs so that the surplus is restricted to be nonnegative. The optimal strategies as well as the value functions are concisely written in terms of the scale function. Numerical results are also given.

1. Introduction

In risk theory, the model of periodic payments has drawn much attention recently. While a majority of the existing continuous-time models assume that dividends can be paid at all times and instantaneously, in reality dividend decisions can only be made at some intervals. Solving the optimal dividend problem under periodic payments is in general difficult. However, thanks to the recent developments of the fluctuation theory, in particular, of Lévy processes, it is getting more tractable.

In this paper, we consider the optimal dividend problem under the constraint that dividend payments can only be made at the jump times of an independent Poisson process. We focus on the dual model (or the spectrally positive Lévy model), which is known to be an appropriate model for a company driven by inventions or discoveries (see, e.g. Avanzi et al., 2007; Avanzi and Gerber, 2008; Avanzi et al., 2011; Bayraktar et al., 2013; Bayraktar et al., 2014; Li et al., 2016; Marciniak and Palmowski, 2016; Yin and Wen, 2013; Zhao et al., 2015). In this context, Avanzi et al. (2014) solved the case with i.i.d. hyperexponential jumps. Our objective is to generalize their results for a general spectrally positive Lévy process with a terminal payoff (penalty) at ruin, and also solve its extension with classical bail-outs so that the surplus is restricted to be nonnegative uniformly in time. Recently, Zhao et al. (forthcoming) studied similar problems where they consider the case with no terminal payoff (penalty) at ruin but with fixed cost for capital injection. For a related problem with Parisian delay, see, among others, Czarna and Palmowski (2014).

In order to solve the problem, we use the recent results given in Avram et al. (forthcoming). As has been already confirmed in Avanzi et al. (2014), the *periodic barrier strategy* is expected to be optimal. Namely, at each dividend-decision time, dividends are paid if and only if the surplus is above some barrier and then it is pushed down to the barrier. The resulting surplus process becomes

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the dual of the *Parisian-reflected process* considered in Avram et al. (forthcoming). Therefore the expected net present value (NPV) of dividends can be computed concisely using the scale function, which enables one to follow the classical "*guess and verify*" technique described below:

(1) In the guessing step, the candidate barrier level b^* is first chosen. Proceeding like in the existing literature (see, e.g., Avram et al., 2007; Bayraktar et al., 2013; Bayraktar et al., 2014; Bensoussan et al., 2005; Hernández-Hernández et al., 2016), b^* (if strictly positive) is set so that the value function becomes "smooth" at the barrier. Differently from the classical dual model as in Bayraktar et al. (2013) where the value function becomes $C^1(0,\infty)$ (resp. $C^2(0,\infty)$) for the case X is of bounded (resp. unbounded) variation (see Bayraktar et al., 2014 for the case there is a fixed cost), we shall see in the periodic payment case that the value function becomes $C^2(0,\infty)$ (resp. $C^3(0,\infty)$) for the case X is of bounded (resp. unbounded) variation.

(2) In the verification step, we first obtain the verification lemma, or sufficient conditions for optimality, and then show that the candidate value function corresponding to the selected periodic barrier strategy satisfies all the conditions. We shall see that its slope is larger (resp. smaller) than 1 at the position below (resp. above) the barrier. This together with the martingales constructed using scale functions completes the proof.

We see that $b^* = 0$ can be possible and in this case the *taking all the money and run strategy at the first opportunity* becomes optimal. As has been observed in Avanzi et al. (2014), this can happen even when (the terminal payoff is zero and) the underlying Lévy process drifts to infinity, while in the classical model this happens if and only if the process drifts to $-\infty$ or oscillates.

In our second problem, we consider the case with classical bailouts, where capital must be injected so that the surplus process remains nonnegative uniformly in time; see Avanzi et al. (2011) and Bayraktar et al. (2013) for the classical case. The objective is to maximize the expected NPV of dividends minus the costs of capital injection. Using the results in Avram et al. (forthcoming), the expected NPV under the periodic barrier strategy can be computed. Again, we select the candidate barrier $b^\dagger$ using the same smoothness conditions described above. The optimality is shown similarly by the verification arguments. In fact, most of the results hold verbatim because the resulting value function admits *the same form* as that for the first problem, except that the barrier level is different.

In both problems, the optimal barrier and the value function can be written concisely using the scale function. In order to confirm the obtained analytical results, we give a sequence of numerical experiments using the phase-type Lévy process that admits an analytical form of scale function, and hence the solutions can be instantaneously computed. We shall confirm the optimality and also analyze the behaviors as the frequency of dividend-decision opportunities increases.

Before closing the introduction, we discuss here the connections with the results in Zhao et al. (forthcoming). The first problem considered in Zhao et al. (forthcoming) is the special case of our first problem with no terminal payoff/cost at ruin. While our paper directly uses the results of Avram et al. (forthcoming) to derive the expected NPV of dividends under the periodic barrier strategy, they obtained it in a different way using the results by Albrecher et al. (2016), which gives the identities for spectrally negative Lévy processes observed at Poisson arrival times. For the selection of optimal barrier and verification of optimality, several results in the current paper (Lemmas 4.2 and 4.4, in particular) are used. The second problem in Zhao et al. (forthcoming) is a variant of our second problem (with capital injection) where they consider the case with a fixed cost for capital injection. With the existence of a fixed cost, the set of capital injection strategies is restricted to be a set of impulse control. As shown in Zhao et al. (forthcoming), their value function converges, as the fixed cost decreases to zero, to that of our second problem.

The rest of the paper is organized as follows. In Section 2, we review the spectrally positive Lévy process and define the two problems to be considered in this paper. In Section 3, we define the periodic barrier strategy (with and without the classical reflection below) and construct the corresponding surplus process. We review the scale function and give the expected NPVs corresponding to these strategies. Sections 4 and 5 solve the first and second problems, respectively. Section 6 gives numerical results and Section 7 concludes the paper. The proofs of the verification lemmas are deferred to the Appendix.

Throughout the paper, $x+ := \lim_{y\downarrow x}$ and $x- := \lim_{y\uparrow x}$ are used to indicate the right- and left-hand limits, respectively. We let $\Delta\zeta(s) := \zeta(s) - \zeta(s-)$ and $\Delta w(\zeta(s)) := w(\zeta(s)) - w(\zeta(s-))$ for any process ζ with left-limits.

2. Preliminaries

2.1. Spectrally positive Lévy processes

Let $X = (X(t);\, t \geq 0)$ be a Lévy process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. For $x \in \mathbb{R}$, we denote by $\mathbb{P}_x$ the law of X when it starts at x and write for convenience $\mathbb{P}$ in place of $\mathbb{P}_0$. Accordingly, we shall write $\mathbb{E}_x$ and $\mathbb{E}$ for the associated expectation operators. In this paper, we shall assume throughout that X is *spectrally positive*, meaning here that it has no negative jumps and that it is not a subordinator. We will assume throughout this work that its Laplace exponent $\psi : [0,\infty) \to \mathbb{R}$, i.e.

$$\mathbb{E}\left[e^{-\theta X(t)}\right] =: e^{\psi(\theta)t}, \qquad t, \theta \geq 0,$$

is given, by the *Lévy–Khintchine formula*

$$\psi(\theta) := \gamma\theta + \frac{\sigma^2}{2}\theta^2 + \int_{(0,\infty)} \left(e^{-\theta z} - 1 + \theta z \mathbf{1}_{\{z<1\}}\right)\Pi(dz), \quad \theta \geq 0, \tag{2.1}$$

where $\gamma \in \mathbb{R}$, $\sigma \geq 0$, and Π is a measure on $(0,\infty)$ called the Lévy measure of X that satisfies

$$\int_{(0,\infty)} (1 \wedge z^2)\Pi(dz) < \infty.$$

It is well-known that X has paths of bounded variation if and only if $\sigma = 0$ and $\int_{(0,1)} z\Pi(dz) < \infty$; in this case, X can be written as

$$X(t) = -ct + S(t), \qquad t \geq 0,$$

where

$$c := \gamma + \int_{(0,1)} z\Pi(dz) \tag{2.2}$$

and $(S(t);\, t \geq 0)$ is a driftless subordinator. Note that necessarily $c > 0$, since we have ruled out the case that X has monotone paths; its Laplace exponent is given by

$$\psi(\theta) = c\theta + \int_{(0,\infty)} \left(e^{-\theta z} - 1\right)\Pi(dz), \quad \theta \geq 0.$$

For the rest of the paper, we assume that

$$\mathbb{E}[X(1)] = -\psi'(0+) < \infty, \tag{2.3}$$

so that the problem considered below will have nontrivial solutions.