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Longevity-linked assets and pre-retirement consumption/portfolio decisions[☆]

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ABSTRACT

We solve the consumption/investment problem of an agent facing a stochastic mortality intensity. The investment set includes a longevity-linked asset, as a derivative on the force of mortality. In a complete and frictionless market, we derive a closed form solution when the agent has Hyperbolic Absolute Risk Aversion preferences and a fixed financial horizon. Our calibrated numerical analysis on US data shows that individuals optimally invest a large fraction of their wealth in longevity-linked assets in the pre-retirement phase, because of their need to hedge against stochastic fluctuations in their remaining lifetime at retirement.

1. Introduction

Despite the relevant and increasing hedging need of pension funds and annuity providers, the market for longevity risk, i.e. the risk of unexpected changes in the mortality of a group of individuals, is not sufficiently liquid yet.

Many reasons may have contributed to undermine a rapid development of the market, such as the lack of standardisation, informational asymmetries, and basis risk. Nevertheless, recent developments provide a sound technology for modelling the systematic randomness in mortality (see e.g. Lee and Carter, 1992), for designing and evaluating hedging instruments (Blake et al., 2006 and Denuit et al., 2007) and for managing longevity risk (Barrieu et al., 2012).

Furthermore, the transfer of longevity risk from pension funds to re-insurers has become more and more common, although on an over-the-counter basis. For instance, the volume of outstanding UK longevity swaps has reached 50 billion pounds as of the end of 2014, with a prevalence of very large deals, such as the 16 billion pounds swap between BT Pension Scheme and Prudential and the 12 billion euros Delta Lloyd/RGA Re index-based transaction. Investment banks have been also active in the transactions. Between 2008 and 2014, alongside reinsurance specialists, JP Morgan, Credit Suisse, Goldman Sachs, Deutsche Bank and Société Générale were involved in longevity deals (Luciano and Regis, 2014).

Longevity-linked products should be of interest to both households and asset managers for at least two reasons: their low correlation to other asset classes (at least in the short run, see Loeys et al., 2007), and their effectiveness in hedging individual investors against the unexpected fluctuations of their subjective discount factors, which take into account lifetime uncertainty (Yaari, 1965; Merton, 1971; Huang et al., 2012). Importantly, from the point of view of individual investors, while traditional insurance products are non-marketable, longevity assets are listed on the market and allow to dynamically hedge against mortality fluctuations.

The aim of this paper is to analyse the optimal consumption and portfolio choices of a consumer/investor subject to longevity risk prior to retirement. The agent can invest in a friction-less, arbitrage free, and complete financial market where both traditional assets (bonds and stocks) and a longevity bond are listed.

An extensive literature has explored consumption and investment decisions when mortality contingent claims are present. In particular, Huang and Milevsky (2008) analyse the decisions of households in the presence of income risk and life insurance. Explicit solutions are also obtained by Pirvu and Zhang (2012) with stochastic asset prices drifts and inflation risk and by Kwak and Lim (2014) with constant relative risk aversion (CRRA) preferences. All these papers consider a deterministic force of mortality, while

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we model it as a stochastic process. We describe longevity risk by means of a doubly stochastic process whose intensity follows a continuous-time diffusion (as in Dahl, 2004). This process may be correlated with all the other state variables. Because of the stochastic mortality, both individuals and annuity/life insurance sellers are exposed to unexpected changes in the force of mortality, implying under or over reserving. For instance, the optimal investment problem of pension funds in the accumulation phase has been studied extensively, for instance by Battocchio et al. (2007) and Delong et al. (2008).

In this paper, we focus instead on the effects of longevity risk and its hedging on individual's consumption and portfolio decisions. The role of longevity-linked assets in investor's optimal portfolio has been addressed first by Menoncin (2008). In this work, we generalise Menoncin (2008) in many directions: (i) the consumer is endowed with a stochastic labour income, (ii) investor's preferences belong to the Hyperbolic Absolute Risk Aversion (HARA) family, allowing for a subsistence level of both consumption and final wealth, which significantly affect the inter-temporal behaviour of the optimal asset allocation, (iii) our numerical simulations take into account mean reverting square root processes for both the interest rate and the force of mortality, which allow for a more realistic framework with respect to the simple Ornstein–Uhlenbeck processes, (iv) we focus on pre-retirement decisions (with a finite-horizon), where the final wealth must reach at least a value sufficient to cover the acquisition of a life annuity.

We consider a fixed deterministic retirement age (in contrast, for instance, with Farhi and Panageas, 2007 and Dybvig and Liu, 2010 who consider an endogenous retirement choice), which coincides with the time horizon of the problem. Horneff et al. (2010) and Maurer et al. (2013) numerically analyse life-cycle portfolio investment problems with longevity risk and focus on the role of deferred annuities and variable annuities respectively. In the context of a life-cycle model, Cocco and Gomes (2012) analyse the demand for a perfect hedge against shocks in the life expectancy of a CRRA agent. They study the optimal investment in a longevity bond, which is akin to the zero-coupon longevity asset that we use in this work. In their numerical simulations, they find that individuals – at old ages and especially approaching retirement – should invest a relevant fraction of their wealth in the longevity asset. Even if our results align with the findings of this literature, we highlight the importance of individual's systematic longevity risk protection in motivating the holding of longevity-linked securities, rather than consumption smoothing after retirement.

Our main contribution consists in providing a closed form solution to the (finite-horizon) problem of an agent prior to retirement, endowed with a general HARA class of preferences when mortality intensity is stochastic. We also provide a calibrated application, which allows to appreciate the magnitude of the investment in longevity-linked products in the optimal agent's portfolio.

Under reasonable stochastic models and calibration for both mortality intensity and interest rate, we find that individuals should optimally invest a relevant fraction of their wealth in a longevity bond. A 60-year old US male, who wants to retire at 65 optimally invests around 70% of his portfolio in the (zero market price of risk) longevity bond and then progressively decreases this share approaching retirement. Demand of the longevity-linked asset is due to hedging motives, as the individual invests in longevity bonds to protect against the fluctuations in his/her discount factor, which accounts for an uncertain lifetime. Because of the stochastic force of mortality, also the amount of wealth that must be invested, at retirement, to obtain a life annuity, varies over time. We explore the sensitivity of our results to both individual and market characteristics, finding that the optimal demand for longevity bonds: (i) is higher for 60-year old US females than for 60-year old US males; (ii) reduces (but very slightly) when the agent displays a more conservative behaviour (either high risk aversion, or high minimum consumption or high final wealth minimum level); (iii) remains positive over the whole horizon even when longevity risk is not remunerated. These last two results are robust for younger agents.

The outline of the paper is the following. Section 2 presents the model setup, while Section 3 describes the individual preferences and the maximisation problem. The optimal consumption and portfolio are found in closed form. Section 4 provides a calibrated application based on US data. Finally, Section 5 concludes, and some technical derivations are left to two Appendices.

2. The model setup

2.1. State variables

On a continuously open and friction-less financial market over the time set $[t_0, +\infty[$, the economic framework is described by a set of s state variables $z(t) \in \mathbb{R}^s$ which solve the following (matrix) stochastic differential equation:

$$\underset{s\times 1}{dz(t)} = \underset{s\times 1}{\mu_z(t,z)}dt + \underset{s\times n}{\Omega(t,z)'}\underset{n\times 1}{dW(t)}, \tag{1}$$

where $z(t_0)$ is a deterministic vector that defines the initial state of the system, $W(t)$ is a vector of n independent Wiener processes,[1] and the prime denotes transposition. The usual properties for guaranteeing the existence of a strong solution to (1) are assumed to hold. The vector $z(t)$ can be divided into two components: the financial state variables $z_f(t)$ and the mortality intensity of a group of individuals, which, as customary in the actuarial literature, are assumed to be homogeneous by cohort, i.e. they belong to the same generation:

$$\underbrace{\begin{bmatrix} \underset{(s-1)\times 1}{dz_f(t)} \\ d\lambda(t) \end{bmatrix}}_{dz(t)} = \underbrace{\begin{bmatrix} \underset{(s-1)\times 1}{\mu_{zf}(t,z)} \\ \mu_\lambda(t,z) \end{bmatrix}}_{\mu_z(t,z)} dt + \underbrace{\begin{bmatrix} \underset{(s-1)\times(n-1)}{\Omega_f(t,z)'} & \underset{(s-1)\times 1}{\mathbf{0}} \\ \underset{1\times(n-1)}{\sigma_{f\lambda}(t,z)'} & \sigma_\lambda(t,z) \end{bmatrix}}_{\Omega(t,z)'} \times \underbrace{\begin{bmatrix} \underset{(n-1)\times 1}{dW_f(t)} \\ dW_\lambda(t) \end{bmatrix}}_{dW(t)}, \tag{2}$$

where $\mathbf{0}$ is a vector of zeros. The diffusion vector $\sigma_{f\lambda}(t,z)$ captures the correlation between the financial state variables $z_f(t)$ and the mortality intensity $\lambda(t)$.

2.2. Financial market

On the financial market n risky assets are traded. Their prices $S(t) \in \mathbb{R}^n_+$ solve the (matrix) stochastic differential equation

$$\underset{n\times 1}{I_S^{-1}}\underset{n\times 1}{dS(t)} = \underset{n\times 1}{\mu(t,z)}dt + \underset{n\times n}{\Sigma(t,z)'}\underset{n\times 1}{dW(t)}, \tag{3}$$

where I_S is a diagonal matrix containing the elements of vector $S(t)$. The initial asset prices $S(t_0)$ are deterministic. Finally, a risk-less asset exists, whose price $G(t) \in \mathbb{R}_+$ solves the ordinary differential equation

$$G(t)^{-1}dG(t) = r(t,z)\,dt, \tag{4}$$

where $r(t,z) \in \mathbb{R}_+$ is the instantaneously risk-less interest rate. We assume $G(t_0) = 1$, i.e. the risk-less asset is the *numéraire* of the economy. The financial market is assumed to be arbitrage free and

[1] The case with dependent Wiener processes can be easily obtained through the Cholesky's decomposition.