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Complete discounted cash flow valuation

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ABSTRACT

This paper concerns discounted cash flow valuation of a company. When the company is in trouble, the owners have an option to provide it with a new capital; otherwise it is liquidated. In the absence of capital outflows and inflows, the company's own funds are modelled by a spectrally negative Lévy process. Within this framework, we look for a strategy of dividend payments and capital injections which maximizes the firm's value. We provide an optimal strategy as well as the corresponding valuation formula. Illustrative examples are given.

1. Introduction

Discounted cash flow (DCF) valuation estimates the fundamental value of a company by the cumulative present values of its future cash flows. A difficulty is that neither the exact values nor the duration of cash flows may be known at the valuation date. The cash flows representing the dividend payments are subject to the firm's dividend policy. The capital injections and their duration depend on the future decision of the shareholders to execute their option to abandon the investment. This "option-like" nature of cash flows poses the biggest conceptual challenge. To understand why, consider the following question: Whether to provide a troubled company with a new capital or not? In practice, the answer depends on whether its value exceeds the bail-out expense. This, however, leads to the following technical difficulty: we need cash flows to determine value, and value to determine cash flows.

How was this problem coped with in the past? An approach popular in corporate finance theory makes the going concern assumption, i.e. that there is no possibility to go bankrupt. This makes the lifetime of a firm infinite and overestimates the company's value. Consequently, a crude correction is usually made either by raising the discount rate or by lowering the expected cash flows. In either case, it is not a trivial question how this adjustment should be made, and ad hoc methods are often used. A detailed review of historical and state-of-the-art methods in corporate finance can be found in Damodaran (2002, 2006) and Koller et al. (2006).

Another approach, popular in actuarial sciences, assumes that the dividend process is a controlled variable and the time of bankruptcy is the first moment when liabilities exceed assets. This essentially simplifies matter as the time of bankruptcy no longer

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depends directly on the value of the company. However, it neglects the possibility of a bail-out and decreases the company's valuation. This approach was proposed independently by de Finetti (1957) and Shubik and Thompson (1959) (for more recent literature, see e.g. Gerber and Shiu, 2004, 2006; Azcue and Muler, 2005; Kyprianou and Palmowski, 2007; Avram et al., 2007; Loeffen, 2008, 2009a,b and Schmidli, 2008).

More recently, an approach coming from option pricing theory has been used to the firm valuation problem (see Dixit and Pindyck, 1996 for an overview of real options theory and Duffie, 2001 or Bielecki and Rutkowski, 2002 for a summary of corporate debt and optimal capital structure applications). In the context of our paper, this approach would typically suggest to find an optimal time to abandon investment, given the dynamics of future cash flows.

The valuation problem can be directly solved by combining the above approaches. A crucial aspect to this is allowing decision-makers to control *both* the cash flows and the moment of shutting down the business. In general, declaring bankruptcy need not be related to the value of the firm. However, it seems reasonable to expect that the company owners' interest is to maximize this quantity. The benefits of this approach are two-fold. First, we obtain a more realistic valuation, thus finding a benchmark to aforementioned valuation methods. Second, we gain insight into optimal corporate decision making. We call this approach a complete DCF valuation.

Two important issues that determine the firm's ability to continue operations are liquidity and solvency. The latter is more important than the former for insurers due to their natural excess of liquidity. On the other hand, liquidity is of primary interest for the banking sector where typically assets have greater duration than liabilities. In this paper, we focus on the solvency issue. For the sake of clarity, however, we will also mention some recent results related to the liquidity approach. How to optimally manage liquidity and solvency in one integrated model is an open problem to the best of our knowledge.

We will assume that the company is subject to a regulatory capital requirement, say $\kappa \geq 0$, which means that it has to keep its own funds above κ. For the insurance sector in the European Union, κ corresponds to Solvency Capital Requirement (SCR) introduced in Solvency II. Roughly speaking, the own funds represent the excess of assets over liabilities. A precise definition, however, depends on accounting standards (see e.g. European Parliament and Council of the European Union, 2009, Chapter VI, Section 3). The excess of the own funds over κ, without dividend payments and capital injections, will be called a surplus and denoted by $\{X_t\}_{t\geq 0}$. Let (a, c) strategy correspond to paying out any excess of surplus over the threshold a and injecting minimal amount of capital required to keep a surplus above zero, until deficit exceeds the threshold c. Denote by $V_{a,c}(x)$ the value of the company under (a, c) strategy, where x stands for the initial surplus.

In this paper, we assume that $\{X_t\}_{t\geq 0}$ is modelled by a spectrally negative Lévy process with the characteristic triple (d, Q, ν). The Wiener part of $\{X_t\}_{t\geq 0}$ is, roughly speaking, related to all balance sheet components of diffusion type. The jump part of $\{X_t\}_{t\geq 0}$ corresponds to discontinuous changes in assets and liabilities. The drift of $\{X_t\}_{t\geq 0}$ contains, among other things, the expected return on assets.

Maximization of the discounted dividend payments minus capital injections, with no possibility to go bankrupt, was studied by Avram et al. (2007) and Kulenko and Schmidli (2008), among others. In the case of a spectrally negative Lévy process and a Cramér–Lundberg process, respectively, they showed that the optimal solution is a barrier strategy.

A corresponding problem with explicit possibility to go bankrupt due to the lack of liquidity was considered by Løkka and Zervos (2008), Décamps et al. (2011) and Hugonnier and Morellec (2015). In the first paper, the underlying process is a Brownian motion and the optimal strategy implies that the bankruptcy happens immediately or at infinity, or as in the de Finetti framework. Décamps et al. (2011) and Hugonnier and Morellec (2015) model the underlying dynamics of the liquid reserves of a bank by a Brownian motion or a Brownian motion with exponentially distributed jumps, respectively. Décamps et al. (2011) set the liquidation value to zero and consider fixed as well as proportional costs. Hugonnier and Morellec (2015) take into account the liquidation value and fixed costs. Both papers consider discrete capital injections. The authors consider strategies that put the liquid reserves strictly above zero every time it becomes non-positive, until bankruptcy occurs. However, a natural class of strategies with the minimal capital injections sufficient to make the firm liquid is ruled out. Thus, there is no need to deal with the local time of the underlying process.

In this paper, we allow the surplus to be put at or above zero whenever it becomes non-positive. The owners may liquidate the firm at any moment and the liquidation value is a random functional of their strategy. We consider proportional costs related to the dividend tax and the costs of capital, respectively. Theorem 4 provides a closed form valuation of a company under an (a, c) strategy in terms of the characteristic triplet of $\{X_t\}_{t\geq 0}$ and the corresponding scale functions. Moreover, under additional assumptions, Theorem 14 states that the (a^*, c^*) strategy, defined in Definition 9, is optimal within the set of all possible strategies.

The results of this paper are partly related to Gajek and Kuciński (2011), who treated the case when $\{X_t\}_{t\geq 0}$ was a Cramér–Lundberg process, and Kuciński (2015). An interesting observation is that the results of the present paper cannot be obtained by a straightforward approximation argument applied to Gajek and Kuciński (2011). The main obstacle appears when the Gaussian component of $\{X_t\}_{t\geq 0}$ is non-trivial.

The remainder of this paper is organized as follows. In Section 2, we formally set up the model. Section 3 contains the valuation formula for (a, c) strategies. Some functionals of surplus at default are also investigated. In Section 4, optimality of the (a^*, c^*) strategy is proven. Heuristic choice of an optimal strategy is given and the properties of the value function corresponding to (a, c) strategies are studied. Additionally, a connection between own funds, DCF valuation and the dynamics of stock returns is outlined as well as the resulting testable implications. Examples containing explicit formulas and comparative statics results on a^* and c^* are provided in Section 5. Conclusions are presented in Section 6. Auxiliary results and proofs are gathered in the Appendix.

2. The model

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space. We assume that the surplus $\{X_t\}_{t\geq 0}$ follows a spectrally negative Lévy process (i.e. a process with stationary and independent increments, and with neither positive jumps nor monotonic paths). Let $\{\mathcal{F}_t\}_{t\geq 0}$ be the underlying filtration (for definition, see Appendix) and assume that there exists a family $\{\mathbb{P}_x\}_{x\in\mathbb{R}}$ of probability measures on $(\Omega, \mathcal{F})$ such that the law of $\{X_t\}_{t\geq 0}$ under $\mathbb{P}_x$ is the same as the law of $\{x + X_t\}_{t\geq 0}$ under $\mathbb{P}$. The law of $\{X_t\}_{t\geq 0}$ is characterized by the Laplace exponent which, by the Lévy–Khintchine formula, can be written as

$$\psi(\theta) = \ln \mathbb{E}\left[e^{\theta X_1}\right] = d\theta + \frac{1}{2}Q^2\theta^2 + \int_{-\infty}^{0} (e^{\theta y} - 1 - \theta y \mathbb{1}_{\{|y|<1\}})\nu(\mathrm{d}y), \quad \theta \geq 0,$$

where $d \in \mathbb{R}$, $Q \geq 0$ and ν is the so-called Lévy measure that satisfies $\nu(0, \infty) = 0$ and $\int_{\mathbb{R}}(x^2 \wedge 1)\nu(\mathrm{d}x) < \infty$. The right-inverse of ψ is denoted by $\Phi(q) = \sup\{\theta \geq 0 \colon \psi(\theta) = q\}$.