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Optimal consumption, portfolio, and life insurance policies under interest rate and inflation risks

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ABSTRACT

This paper solves the optimal life insurance, consumption, and portfolio decisions of a wage earner before retirement under interest rate and inflation risks. The wage earner's preferences are represented by the stochastic differential utility, which separates the coefficient of relative risk aversion from the elasticity of intertemporal substitution (EIS). The wage earner's life insurance demand is affected by the volatile interest rates and inflation. The optimal life insurance demand decreases with the level of nominal interest rates. Under an assumption of deterministic nominal income, the demand for life insurance would not be affected by the level of inflation. However, if the wage earner's income is indexed to inflation, the life insurance demand would increase with the level of inflation. Furthermore, under investment opportunities with greater volatilities, wage earners who optimally allocate their wealth to the financial market benefit more from financial investments and cut their demand for life insurance. An analysis of EIS and risk aversion on life insurance demand shows that the demand for life insurance over the planning horizon increases with the measure of relative risk aversion but decreases with EIS. Optimal consumption is affected by the insurance premium load and the direction depends on the size of EIS relative to unity.

1. Introduction

Wage earners with bequest motives purchase life insurance to protect their dependents against the loss of their human capital, i.e., the present value of future labor income, in the event of premature death before retirement. Following Hakansson (1969) and Richard (1975), many researchers extend the intertemporal consumption–portfolio choice model to incorporate the optimal demand for life insurance under an uncertain lifetime (see, e.g. Babbel and Ohtsuka, 1989; Huang and Milevsky, 2008; Pirvu and Zhang, 2012; Pliska and Ye, 2007). However, in these intertemporal optimization models, the effect of intertemporal substitution on life insurance demand does not derive as much attention as it deserves. One of the possible reasons is that most existing models are established with the power utility function in which the elasticity of intertemporal substitution (EIS) is represented by the inverse of the coefficient of relative risk aversion; as a result, the effects of EIS and risk aversion are entangled.

In this study, a continuous-time model is proposed to analyze a wage earner's optimal consumption, portfolio, and life insurance strategies before retirement. It is assumed that the wage earner's lifetime utility includes the utility of consumption when alive and the utility of bequest when a premature death occurs before retirement. The legacy received by the wage earner's dependents is composed of the payment of life insurance and the wage earner's financials savings upon death. To disentangle the effects of risk aversion and EIS on optimal life insurance purchase, we employ the stochastic differential utility (SDU) proposed by Duffie and Epstein (1992) to represent the wage earner's preferences. SDU is the continuous-time limit of the recursive utility studied by Kreps and Porteus (1978) and Epstein and Zin (1989) and nests the time-additive power utility as a special case.

Furthermore, since the problem of optimal consumption, portfolio, and life insurance is a long-term financial planning problem, it is more realistic to take the uncertainties of investment opportunities into consideration. In this paper, we consider the risks of interest rates and inflation in the environment. The exact solution of the optimal strategies under SDU with stochastic

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investment opportunities is obtained in two special cases: the case of unit EIS and the case of time-additive power utility. In a more general case with EIS different from unity, we employ the log-linearization method introduced by Chacko and Viceira (2005) to find an approximate solution of the problem.

Under the assumption of stochastic interest rates and inflation, we find that the optimal portfolio consists of a self-financing and a replicating portfolio. The self-financing portfolio is the portfolio as if there is no human capital. It includes a speculative demand determined by the risk-return trade-off of risky assets and a hedging demand which hedges against the uncertainties of interest rates and inflation. The replicating portfolio is referred to as the portfolio which replicates the returns on the wage earner's human capital to hedge against the risk of human capital.

The optimal portfolio of wage earners with low risk aversion is highly leveraged. For the purpose of speculation, it takes a short position in inflation-indexed bonds and invests heavily on nominal bonds and stocks. When the wage earners are highly risk averse, the portfolio is balanced. The demands for nominal bonds and stocks fall and the demand for indexed bonds rises to hedge against the inflation risk.

As to the optimal life insurance demand, we investigate the effects of interest rates and inflation. We find that the optimal proportion of human capital being insured would decrease with the level of nominal interest rate. This is because that the wage earner's human capital is the discounted present value of his or her future labor income and is equivalent to a nominal coupon bond. A high level of interest rate would reduce the value of human capital and thus the demand for life insurance. Under an assumption of deterministic nominal income, the wager earner's human capital is equivalent to a nominal coupon bond and the optimal proportion of human capital being insured is not affected by the level of inflation. However, if the wage earner's income is indexed to inflation, the value of human capital would be equivalent to an inflation-indexed coupon bond. In this case, a high inflation rate would increase the value of human capital and therefore the demand for life insurance.

Taking the uncertainties of interest rates and inflation into consideration, we analyze the effects of interest rate volatility and inflation volatility on optimal life insurance demand. The results show that the wage earners who optimally allocate their wealth to the financial market would earn more risk premium when interest rates and inflation are more volatile. The risk premium would raise the growth rate of the wage earner's financial savings and, as a result, would decrease the wage earner's demand for life insurance.

An analysis of EIS and risk aversion on life insurance demand shows that EIS has no cross-sectional effect on optimal life insurance demand, i.e., with certain amounts of human capital and financial savings, the wage earner's demand for life insurance would increase with the extent of risk aversion while being independent of EIS. This result is consistent with the findings of previous studies based on the power utility. However, this cross-sectional irrelevance of EIS with life insurance demand does not necessarily imply that the overall expenditure on life insurance would not be affected by EIS. In fact, at any specific time, the life insurance demand would be affected by the amount of the wage earner's financial savings over the planning horizon since the life insurance payment and the financial savings act as substitutes for each other. For individuals endowed with the same initial wealth and wage income, their financial savings could be quite different depending on how they allocate their resources.

In this study, we examine the intertemporal effects of EIS and risk aversion on life insurance demand. The wage earner with a higher EIS purchases less life insurance over the planning horizon. This is because with a higher EIS, the wage earner is more willing to substitute the future for current consumptions. This leads to a higher level of financial savings over the planning horizon and, as a result, a lower life insurance demand. Next, the wage earner with a higher risk aversion purchases more life insurance over the planning horizon. As indicated by earlier research, the wage earner who is more risk averse would cross-sectionally purchase more life insurance to hedge against the loss of human capital in the event of premature death. Moreover, a more risk-averse wage earner would invest less in the risky asset and relinquish the opportunities to collect the risk premium. In the long run, this would result in a lower level of financial savings over the planning horizon and increase the demand for life insurance.

The numerical examples in this study show that the effect of EIS is weaker with a higher risk aversion and the effect of risk aversion is stronger with a higher EIS. We also show that the optimal consumption–wealth ratio is affected by the load of insurance premium. The direction of this effect is decided by the level of EIS. When EIS is smaller than unity, the optimal consumption–wealth ratio would increase with the load of life insurance premium. When EIS is greater than unity, the optimal consumption–wealth ratio would decrease with life insurance premium load.

2. Related literature

The Bulk of the published literature has been devoted to the research of the demand for life insurance from an individual's perspective since the pioneering work of Yaari (1965), who shows that the intertemporal optimization problem with an uncertain lifetime could be transformed into an equivalent one with a certain horizon. Hakansson (1969) uses the discrete-time dynamic programming approach to solve the optimal consumption, investment, and life insurance strategies in one model. Richard (1975) extends the seminal work of Merton (1971) to solve the optimal decisions of consumption, portfolio choice, as well as life insurance with a continuous-time model. Campbell (1980), who uses a single-period model, solves the demand function for life insurance explicitly in terms of the individual's risk aversion, intensity for bequests, and the insurance company's loading charge. Pliska and Ye (2007) and Zhu (2007) analyze the effects that a wide array of economic parameters, such as risk aversion, subjective time preferences, risk-free rate, and insurance premium load, have on optimal life insurance demand. Huang and Milevsky (2008) and Huang et al. (2008) solve the optimal portfolio and life insurance decisions by focusing on the correlation between the dynamics of human and financial capital. More recently, Duarte et al. (2014) extend the model of Pliska and Ye (2007) to solve the optimal insurance, consumption, and portfolio rules with multiple risky securities. Pirvu and Zhang (2012) and Kwak and Lim (2014) solve the optimal consumption, investment, and life insurance problem with stochastic investment opportunities. Pirvu and Zhang (2012) investigate the effect of the stochastic market price of financial risk on optimal insurance; and Kwak and Lim (2014) analyze the effect of inflation risk on life insurance demand.

Most of these existing works are established under the widely used power utility function and consistently show that the optimal life insurance demand would increase with the measure of relative risk aversion. With the power utility function, EIS is represented by the reciprocal of the coefficient of relative risk aversion; as a result, the effects of EIS and risk aversion on optimal policies are entangled. To disentangle the effects of risk aversion and EIS, we use SDU proposed by Duffie and Epstein (1992) to represent the preferences of the wage earner. SDU has been used to investigate the optimal consumption–portfolio problem in a financial context (see, e.g. Bhamra and Uppal, 2006; Chacko and Viceira, 2005; Chou et al., 2011; Schroder and Skiadas, 1999; Svensson, 1989). As pointed out by Svensson (1989), EIS affects the consumption–saving decision but it has no effect on the optimal portfolio decision under constant investment opportunities. Under stochastic investment opportunities, Bhamra and Uppal (2006)